I. INTRODUCTION AND CONCLUSION

This article summarizes the influence of slurry rheology on the grinding throughput of batch laboratory and continuous plant-scale mills operating in open and closed circuit configurations. The results of controlled changes in percent solids, particle size, slurry temperature, and viscosity control chemicals will be demonstrated. This paper is a condensation of portions of a general reference paper by Klimpel to appear in late 1982 or early 1983 in Mining Engineering. Readers interested in more information on the relationships between fundamental breakage concepts and slurry rheology should consult the above reference.

An underlying rheological explanation is offered which ties the type and magnitude of particle breakage occurring in the mill with the corresponding rheological character that the slurry was exhibiting at some given set of operating conditions. More specifically, dilatant slurries show first-order breakage, pseudoplastic slurries also demonstrate first-order breakage, but at a higher rate, and pseudoplastic slurries with significant yield values show slower non-first-order breakage.

A general observation is the need, from a maximum throughput basis, of tumbling media mills to operate on as thick a slurry basis as is possible that still offers a low enough viscosity to keep grinding first-order in nature. A series of industrial scale operating guidelines are presented which have proven useful in increasing the net production capability of grinding circuits. Specifically, the influence of percent solids, adding fines, decreasing temperature, the use of viscosity control chemicals, and the role of classification in closed circuit milling will be discussed. A general observation on both open and closed circuit operation was that it is necessary to have the slurry in the grinding mill itself be in a pseudoplastic rheology region in order to achieve maximum net throughput. Typical interactions of grinding mill rheology and classifier operation will be presented. The use of the concepts of this paper on an industrial scale have proven valuable to the operating mineral/coal processing firms involved in the over-all testing program.

II. SUMMARY OF VISCOSITY EFFECTS IN LABORATORY GRINDING STUDIES

In previous laboratory studies [Klimpel and Manfroy (1978), Klimpel (1980), Katzer et al (1981), Klimpel and Austin (1982)], it was clearly shown that there is a consistent pattern of change in specific rates of breakage of both mineral and coal slurries as slurry fluidity is changed. For example, using the net production rate of material less than some specified size (e.g., kg/hr of minus 75 μm) as an index of mill production in a standard batch mill test (with a given feed material, feed size, mill, and mill conditions such as constant time of grind), the following facts were readily established:

1. The normal range of low slurry density, low viscosity (Region A) gave no variation in mill production. The measured rates of break-
age exhibited normal first order grinding.

2. Grinding of a somewhat higher viscosity slurry (Region B) could give increased production. The higher viscosity was obtained by increase in slurry density and/or by size distribution control. Measured rates of breakage were again first order but were somewhat faster than in (1).

3. Too high viscosity (Region C) gave decreased production. This was associated with non-first order grinding, that is, a slowing down of grinding rates as the grinding proceeded, due to the production of fines giving increased slurry viscosity.

4. Certain water-soluble chemicals allowed the effect in (2) to be extended to higher production rates before the effect of (3) became controlling.

Figure I illustrates graphically the four conclusions just outlined. The Brookfield RVT viscosimeter with a T bar and helipath stand is well-known and is useful for giving qualitative comparative data on slurry viscosities.

In order to understand better the grinding behavior exhibited in Figure I, it is valuable to use the concepts of first-order breakage rates and primary breakage distribution [e.g., Austin (1971)]. The complete definitions, experimental procedures involved, and use in mill circuit design/analysis have been described elsewhere [e.g., Austin et al (1982)] and will not be repeated here. Briefly stated, the determination and comparison of these two parameters under varying operating conditions (e.g., percent solids) allows one to make a more complete analysis of data and even more importantly, allows for logical predictions of measured laboratory effects in larger scale industrial use to be made.

The rate of breakage of a given size range of particles (e.g., a $\sqrt{2}$ screen interval) is proportional to the amount of that size present when grinding is first-order. Thus rate of breakage of size fraction $\sim j = S_j w_j(t)W$ where $S_j$ is the specific rate of breakage (fraction per unit time) of size $j$, $W$ is the mill hold-up, and $w_j(t)$ is the weight fraction of size $j$ material at grinding time $t$. Thus, if the starting feed contained $w_j(0)$ as the top size, $dw_j(t)/dt = -S_j w_j(t)$

$log w_j(t) = log w_j(0) - (S_j t/2.3)$. Measuring the disappearance of material from this size as a function of time, using log-linear plots, will directly indicate three important factors: (1) if the plot is linear, the size fraction $j$ is breaking in a first-order manner; the negative slope gives the $S_j$ value; (2) the magnitude of $S_j W$ is a direct indication of grinding throughput; (3) if the plot is not linear but flattens as grinding proceeds (e.g., as fines build-up and viscosity increases), then a slowing-down of breakage is occurring.

The suite of fragments produced by breakage of a given size without further rebreakage of the fragments is termed the primary breakage distribution. Numbering size intervals from 1 for the largest size, 2 the next size interval, etc., the primary breakage distribution is represented by $b_{ij}$, that is, the fraction of just broken $j$ material which falls into smaller size interval $i$. $B_i, j$ represents the same information put on a cumulative basis of material broken from size $j$ to size $i$ or smaller

$$B_{i,j} = \sum_{k=n}^{i} b_{k,j}$$

Experimental data on $B_{i,j}$ is fitted to the empirical function
Figure II illustrates graphically the nature of the specific rate of breakage, \( S_j \), and primary breakage distribution as a function of the three slurry viscosity regions indicated in Figure I. Numerous quantitative examples of Figure II were determined in the study for a variety of ores and coals [e.g., Klimpel and Austin (1982) and Klimpel (1982b)]. The influence of percent solids (viscosity) on the specific rate of breakage shows the trends discussed earlier. Regions A, B, and B' demonstrate first order breakage, Regions B and B' give higher rates of breakage than Regions A and C, Regions C and C' show slower non first-order breakage, and grinding with viscosity control chemicals shows no advantage in Region A, but an increase over the no chemical case in Regions B and C. It is interesting to note the need, from a maximum throughput basis, of tumbling media mills to operate on as thick a slurry basis as is possible that still offers a low enough viscosity to keep grinding in a first-order manner. This tendency was not fully recognized until this study, and is a very basic character common to tumbling media mills.

Figure II shows that the measured \( B_{i,j} \) values for breakage in high density slurries have a relatively finer primary fragment distribution (a higher \( \phi \) and smaller \( \gamma \)) than for lower density slurries. Related laboratory tests have also shown that in the grinding of homogeneous materials, little change occurs in the relative manner in which different sized particles are selected for breakage as a function of slurry density. Thus, for example, when the \( S_j \) values are described by \( S_j = a(x_j/x_1)^\alpha \) the \( \alpha \) remains essentially constant and only the \( a \) varies with viscosity region [Klimpel and Manfroy (1978), and Klimpel and Austin (1982)].

With regard to the identification of suitable viscosity control chemicals (see patents of Manfroy and Klimpel) to act as grinding aids, a number of necessary conditions were established [Klimpel (1980)]. Most of the chemicals identified were low molecular weight water soluble polymers. Briefly stated, the required conditions are (1) the chemical must adsorb on enough of the solid surfaces available in the ore/coal being ground so as to affect slurry viscosity; (2) the slurry viscosity must be high enough so that use of the chemical can help reduce or control slurry viscosity; (3) the chemical must be consistent in its ability to lower viscosity as a function of varying chemical concentration, \( \text{pH} \), water quality (especially in the presence of divalent, trivalent ions), and amount of shear present; (4) the chemical must be non-toxic and degradable; (5) the chemical must not adversely affect downstream operations such as flotation, thickening, pelletization, etc., and (6) the use of chemical must be economically viable in grinding operations. When any one of the above conditions is seriously violated, the use of grinding aid on a commercial scale will not be feasible.

Not surprisingly, it has been found that 2 to 10 times more chemical may be required in batch laboratory tests than in corresponding continuous mill tests because of the higher surface area changes involved in batch grinding tests. As will be discussed in the next section on basic slurry rheology characterization, there have been a few ore/coal slurries that have not given increased throughput with chemical usage, even though all of the previously listed chemical conditions were satisfied. This has to do with the nature of the
rheological behavior of the slurry itself and is not directly a chemical problem.

In addition, with the use of grinding aid chemicals, some positive downstream influences have been found including increases in observed flotation plant recoveries [Klimpel (1980)], and improved separations in heavy media operations and in classification [e.g., Klimpel (1982a)]. The use of chemicals to improve grinding in plant circuits was not a problem in thickening, etc., as long as the dosage levels were kept within a range normally associated with grinding mill operation (e.g., <0.5 kg GA-4272*/tonne of dry ore feed). In batch grinding, the use of viscosity control chemicals will help to grind a given mass finer in some specified time while in continuous grinding environments, chemicals can be used to grind a given feed rate finer or to increase feed rate ground to a given particle size. The ability to achieve these results depends in part on the equipment involved, ore type, mill operating conditions, as well as the chemical requirements just discussed.

III. FUNDAMENTAL RHEOLOGICAL CHARACTERIZATION OF BREAKAGE IN LABORATORY MILLS

Despite the apparent success and consistency of the type of data reported in the previous section, it became obvious during the early stages of the research program that a more fundamental rheological explanation was required. For example, correlations of throughput versus percent solids as developed in this study or by plant operating personnel are useful only over very limited ranges and cannot be easily extrapolated because of their inherent assumption of constant particle size, etc. Such assumptions are only valid for continuous mills operating on consistent feeds in a consistent manner. It is not difficult, for example, to show in a batch laboratory test that even for a fixed value of percent solids, rather small changes in particle size make-up (especially of the fines) contributing to the solids can make very large differences in the throughput rate. These throughput differences due to particle size can sometimes overwhelm the influence of only changing percent solids. This type of problem seriously limits the generality of most plant level percent solids/throughput correlations that have been developed. With a few ores and coals, no Region B could be identified. With a few other ores, the influence of chemicals was minimal despite the apparent satisfaction of all desired chemical characteristics such as adsorption, etc. What then, is a more basic explanation?

The answer to providing a more consistent explanation and prediction of grinding behavior was provided by accurately measuring quantitative slurry rheology data. This was done using a Haake RV3 rotational viscosimeter, which is not as convenient as the Brookfield, especially for routine plant use, and requires special experimental procedures. However, the data from the rotational viscosimeter can be obtained under sufficiently controlled operating conditions and known geometric configurations to enable plots of shear stress versus rate of shear to be constructed for comparison with the possible types of slurry behavior shown in Figure III.

Briefly stated, at a given time, all mineral/coal slurries will exhibit one of the shear-strain characters outlined in Figure III [see Bird et al (1960) and Wasp et al (1977)]. Pseudoplastic slurries may or may not have a yield value and when shear stress $T$ is plotted versus
shear rate \( \Delta \), a curve results that has a decreasing slope with increasing rate of shear, and generally approaches a limiting slope at higher shear rates. Dilatant slurries exhibit the opposite behavior; that is, an increasing slope of \( \gamma \) versus \( \Delta \). A common method of mathematically describing both of these types is the Ostwald-de-Waele or Power Law Model: 
\[
\tau = K\dot{\gamma}^n
\]
where \( K \) and \( n \) are constants for a particular slurry. The constant \( K \) is referred to as consistency; the higher the value of \( K \), the more viscous the slurry. The constant \( n \) is called the flow index, which is a measure of the degree of departure from Newtonian behavior (\( n = 1 \)), \( n > 1 \) gives pseudoplastic behavior, and \( n < 1 \) dilatant behavior.

Detailed rate of breakage tests were carried out on single size fractions as outlined earlier [Klimpel (1982b)]. For each test condition corresponding to the rate of breakage tests, rheological characterization evaluations were performed. Controlled changes in percent solids, fineness of grind, and temperature were made during the tests.

Interpreting all of the data collected, the following conclusions were drawn [Klimpel (1982b)]:

1. Many coals and mineral slurries exhibit dilatant character at relatively low slurry densities, less than 40-45% solids volume for typical size distributions; closely sized solids give more dilatant character than broad distributions.

2. In this dilatant region, grinding is first-order, and absolute rates of breakage SW do not vary during the grinding or from one slurry density to another. This is Region A of Figure I and represents normal wet grinding practice.

3. Increasing slurry density causes a trend toward pseudoplastic behavior. At a given slurry density, more pseudoplastic character can be developed by increasing the solids packing efficiency (by adding a proportion of fines or controlling the size distribution, by the use of a bulk thickening agent, or by the use of chemicals to modify viscosity).

4. When a slurry exhibits pseudoplastic behavior without a yield stress, then grinding is first-order with higher absolute rates of breakage SW than in corresponding dilatant systems. This is Region B and B' of Figure I and represents the most efficient wet grinding practice.

5. Grinding aid chemicals that work best in practice are those which maintain pseudoplastic behavior in the slurry without associated yield stress, or which reduce the yield stress in a dense pseudoplastic slurry.

6. When grinding is performed on a very dense slurry, the yield stress increases rapidly and leads to non first-order breakage with a slowing-down of breakage rates. This is Region C and C' of Figure I.

There are at least four controllable factors which decide the rheological character of a slurry: (1) slurry density (percent solids); (2) particle size distribution; (3) chemical environment; and (4) slurry temperature. The second factor has two interrelated facets: the shape of the particle size distribution (which controls the packing behavior of the solids) and the fineness of the distribution (finer particles increase interparticle forces and increase viscosity). As has been indicated earlier, during a given grinding test it is possible for all four of the above factors to change.
However, regardless of the particular settings of the four factors in a given test, if the resulting rheological character is either dilatant, pseudoplastic, or pseudoplastic with yield, the rate of breakage associated is correlated with the current rheological character.

It is obvious, for example, in batch grinding tests run at constant percent solids, that factors (2) and (3), where appropriate, are changing during grinding because the size distributions are obviously changing and because the production of fresh surface area will take up unadsorbed chemical. Thus, the corresponding rheological character change in batch tests with increasing grind time would be dilatant to pseudoplastic to pseudoplastic with yield. The degree to which this transformation occurs depends on the changing setting of the four factors over grinding time. In continuous grinding tests, one might logically expect that dramatic changes in any one of the four factors will be less likely to occur. However, it will be shown in later sections of this paper, that continuous mill operations offer some unique opportunities to take advantage of possible rheological transformation by more direct operational control of the settings of the four factors.

One extra observation noted in the rheological studies was the variability in the location and extent of Region B (pseudoplastic behavior) for the various coals and ores tested. With regard to data of the form of Figure I, the location of Region B was usually in the region of 45-55 percent solids by volume and was of the extent of zero to eight percent (2 to 11 percent with chemical addition). The corresponding increase in net production ranged from zero to 10 percent in Region B and zero to 21 percent in Region B'. When Region B is small or zero (no pseudoplastic character is exhibited) no increase in production will be observed and the use of chemicals is often marginal. There are several reasons for some materials exhibiting this quick transformation from dilatancy to high yield values (often at surprisingly low percent solids by volume such as 30%). One condition identified was for materials containing high levels of viscosity producing elements such as carbonates, clays, etc. A second condition documented was for materials that exhibited unusually fine primary fragment distributions (fine $B_{ij}$ curves). Such a slurry developed a yield value quickly during grinding due to the rapid build-up of fines.

IV. CONTROLLED RHEOLOGY TESTS IN INDUSTRIAL SCALE OPEN CIRCUIT OPERATIONS

In this section, the results are presented of a series of controlled rheology tests run on an open circuit rod mill operating on a copper slurry. The rod mill was an overflow discharge mill of dimensions 2.5m by 4.25m operating at 70% of critical speed on an average feed rate of 100 tonnes per hour, a rod loading of 36% (7.5 cm make-up rod size), and a feed size of approximately 100% <2.5cm, 50% <6 mesh U.S., 20% <35 mesh U.S., and 10% <200 mesh U.S. Special arrangements were made to keep operating conditions, including ore character, as constant as possible during the tests. Changing the percent solids by changing water was the major operating variable. The total energy draw of the mill was essentially constant during all of the runs in slurry Regions A, B, and B'. Only at very high slurry densities (Region C) was a drop in energy draw noted. Rheological characterizations were also performed during the tests in order to
identify slurry rheology behavior at any desired point in the test program so as to be able to correlate plant results with the laboratory data. Four basic factors were studied including variation in percent solids, the addition of chemical GA-4272, the addition of fines to feed, and the influence of slurry temperature.

Figure IV summarizes typical data collected on the basis of net production less than 35 mesh versus percent solids by weight. The influence of percent solids (with and without chemical) on the normal feed and feed slurry temperature of 22°C is certainly similar in net production and viscosity response to that of Figure I. This is an example where simple optimization of water use can lead to significant plant throughput increases. In similar tests run on rod mills at other plants, however, the feed size was often so coarse that it was not possible to achieve Region B (pseudoplastic slurry character). This was primarily due to the fact that the percent solids could simply not be raised high enough with normal water control procedures to safely operate in Region B. In this environment, the rod mill will operate only in Region A and optimizing water control will produce, at best, only marginal increases in throughput. As explained earlier, the use of a viscosity control chemical in Region A is also not warranted.

In order to more fully demonstrate rheological influences, the same rod mill was then operated with 10% of the feed rate being supplemented by minus 400 U.S. mesh ore. As can be seen from Figure IV, a similar shape of net production curve results as in normal operation, but the maximum throughput (hence the pseudoplastic Region B) now occurs at a lower percent solids. While there is little throughput advantage to adding fines to the feed in this particular test, the addition of fines to rod mills is an important concept. This is true because Figure IV, for example, demonstrates that those rod mills operating on coarse feeds which cannot normally reach rheological Region B by water control alone, can be forced into this region (hence giving higher net throughputs) by the introduction of fines from some other point in the overall process. Several tests at various plants were run to verify this very important concept. The implications of the influence of fines on rod mill performance with regard to crusher operating guidelines is obvious. Crushing operations that produce few fines can actually be quite detrimental to optimal rod mill performance.

Figure IV also illustrates the influence on net production of a lower feed slurry temperature of 8°C. It is obvious that the net production is lower, but equally important, from a control viewpoint, is that the value of percent solids in which Region A is transformed to Region B is lower than normal operation. Very similar results to Figure IV were also generated on rod mills grinding coal, and on open circuit ball mills grinding ores.

V. RHEOLOGY TESTS IN CLOSED CIRCUIT OPERATIONS

Using the same copper ore as in the previous section, a series of tests were also run on a ball mill operating in closed circuit with a single 24 inch hydrocyclone. The mill was an over-flow discharge type of dimensions 3.0m by 5.0m operating at 80% of critical speed on an average feed rate of 200 tonnes per hour, a ball loading of 38% (4.0mm and 2.5mm make-up ball sizes), and a feed size of approximately 100% <35 mesh U.S., 50% <30 mesh U.S., and 20% <200 mesh U.S. As in the rod mill test, the energy draw of the mill was essentially constant during all of the runs.
As in the previous section, special care was taken to maintain consistent operating conditions and rheological characterizations were run. The basic factors studied in the closed circuit tests were variation in percent solids, the addition of chemical GA-4272, the addition of fines to feed, and the influence of classifier efficiency.

Figure V summarizes typical data collected on the basis of net production of the total circuit less than 200 mesh U.S. versus percent solids by weight. As in the rod mill tests, the influence of percent solids (with and without chemical) on the normal feed is similar in net production and viscosity response to that of Figure I. In all of the secondary grinding closed circuit tests run on various ores and coals at various plants, it was generally not difficult to transform slurry rheology from dilatant to pseudoplastic to pseudoplastic with yield merely by adjusting water control. This control can be done by increasing the percent solids by increasing solids feed rate and holding water addition rate constant and/or decreasing water feed rate holding solids feed rate constant. The first method is most useful for increasing the circuit solids throughput while still producing a relatively constant fraction of solids less than some specified size, while the second method is better suited for grinding a given solids feed rate finer than normal [Klimpel and Austin (1982)]. Also, in closed secondary grinding circuit operations, the addition of fines to the make-up feed was not generally advantageous, and thus is not included in Figure V. The exceptions to this observation were primary closed circuit ball mills and/or autogeneous circuits operating on relatively coarser feeds than normally associated with secondary ore grinding circuits.

One of the unexpected results of closed circuit testing was the rheological influence on mill grinding rates of changes in classifier efficiency. As percent solids were being varied under constant feed solids, it became obvious in many of the tests that the classifier (either hydrocyclone or screen type) size separation characteristics were changing with slurry rheology changes. The change of hydrocyclone performance with percent solids variation has been documented previously [e.g., Lynch (1977)]. Also, in normal hydrocyclone operations, it is well known that an increase of flow rate to hydrocyclones will cause a decrease in the \( d_{50} \) value of separation. The influence of viscosity control chemicals, added to the make-up circuit feed, on classifier performance was also quite noticeable. Figure VI gives a Tromp curve example of this effect using GA-4272 on a stand-alone hydrocyclone operating on a copper ore. In general, it was found [see Klimpel (1982a)] that the use of viscosity control chemicals on hydrocycloning increased slightly the apparent solids by-pass fraction and made the sharpness of separation (Sharpness Index S.I. = \( d_{25}/d_{75} \)) more ideal (higher S.I. value) while maintaining or increasing the \( d_{50} \) value (even under conditions of high cyclone feed rate and/or high cyclone feed percent solids).

It was further identified after detailed testing, that the classifier in closed circuit operations can, and does in many circuits, act as a rheological control device on the grinding mill itself that is more important than control of the make-up feed percent solids to the over-all circuit. The classifier can change mill rheology by changing the particle size being recirculated back to the mill, as well as by influencing the circuit water balance. For example, if the classifier
(for whatever reason) presents too coarse a recirculated product (and/or with too much water), it can become very difficult to move the mill rheology out of Region A (dilatancy), even with large decreases in water addition to the make-up feed. In this environment, the net circuit throughput is relatively insensitive to operating conditions and is not operating in a maximum throughput condition. On the other hand, if the classifier (again for whatever reason) presents too fine a recirculated product (and/or with too little water), the mill rheology will have a tendency to be in Region C (slurry yield value present). This is also not an optimal operation from an over-all circuit throughput basis, although the use of viscosity control chemicals will generally help.

The maximum throughput obtainable from any of the closed circuits tested was when the grinding device itself was operating in Region B (pseudoplastic slurry). This is an important conclusion and, in order to achieve mill operation in Region B, often requires a delicate balance of water control and classifier equipment setting. Operating a closed circuit mill in region B or B' can typically give throughput increases of from four to 20% over operating in Regions A and C. The use of viscosity control chemicals makes Region B to B' larger (in terms of percent solids range, for example) and the effect of Region B to B' more pronounced (higher throughput of Region B' over Region B by two to 12%). It was also found that maintaining a mill in Region B may not be easy on a continuous operating plant schedule due to unavailability of appropriate plant online viscosimeters and, in some plants, large variability in ore type, feed size, and water supply. Even in this latter environment, however, if the circuit can be operated at least a portion of the time with the mill in Region B or B', the increase in net throughput will warrant the extra effort.

Another type of test sequence run on the same closed circuit ball mill circuit was to deliberately vary the hydrocyclone classifier size separation efficiency. Figure V also gives the net circuit production of minus 200 mesh material as a function of percent solids for three different separation efficiencies of the hydrocyclone including the normal operating case \( (a = 0.25, d_{50} = 185, S.I. = 0.30) \) and the case with chemical \( (a = 0.27, d_{50} = 190, S.I. = 0.56) \). The smaller \( d_{50} \) case over normal operation \( (a = 0.28, d_{50} = 176, S.I. = 0.49) \) was achieved by decreasing the vortex finder diameter. The influence of recirculating a finer product back to the mill due to the smaller \( d_{50} \) value is evident in the earlier occurrence of pseudoplastic slurry character in terms of percent solids. This type of test was repeated on a variety of closed circuit grinding operations. It was generally found that the mechanical setting and operation of the classifier (as reflected by \( a, d_{50} \), and S.I.) will have a major impact on where (from a percent solids viewpoint, for example) the conversion from dilatant to pseudoplastic to a significant yield slurry will take place. However, in most of the circuit tests run, it was not possible to predict this rheology conversion a priori, and thus, the corresponding determination of maximum net circuit throughput depended on an organized plant testing program. This is unfortunate, as laboratory tests were capable of predicting rheology conversion in open-scale mills, but the addition of the classifier makes such predictions too unreliable. This points out the necessity of developing a better understanding of classifier (especially hydrocyclone) operation in
general and rheology effects in classification, more specifically.

* a patented grinding aid product of The Dow Chemical Company especially designed for sulfide ore processing. Formerly denoted as XFS-4272.

VII. REFERENCES

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