A COMPUTER METHOD FOR THE ON-LINE BALANCING PROBLEM. APPLICATION TO A GRINDING - CLASSIFICATION CIRCUIT

INTRODUCTION

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One of the basic problem in process analysis is that of calculating material balances on a single-unit assembly, a set of units or an entire installation. The balance is fundamental because satisfactory evaluation of the performance index of the process under investigation depends on it.

The problem of a material balancing lies in calculating the most probable real magnitudes to be used to verify the equation of the system; this calculation is generally performed by using a least square adjustment.

Following a brief description of the process under consideration (a grinding classification circuit), the study shows the calculations to be performed in the case of a decentralized material balance equilibration. Then a practical point of view outlines some numerical results.

While the method was developed and presented here with particular reference to a grinding circuit, it may equally be applied to other types of problems without requiring major modification.

1 - DESCRIPTION OF THE PROCESS

The pilot plan of the "Laboratoire d'Automatique et de Recherche Appliquée" at the "Centre de Recherches sur la Valorisation des Minerais" of Nancy (France) is composed of (Figure 1):

- a ball mill (4) with its feed circuit (conveyor-belt, happer),
- a sump (8) to collect the output pulp of the ball-mill,
- a pump which control the flowrate of the input hydrocyclone;
- an hydrocyclone to classify the ore particles.

The figure 1 presents these elements and their connections; the positions of the sensor are also indicated (electromagnetic flow-meters for water, weight-gauge for ore, gamma-gauge for density).

Figure 1 - Grinding-classification process.

In fact the points marked as 5 and 11 are not equipped with sensors and the measurements at these points are manually obtained.
In the foregoing text $Q$ denotes a flow, $d$ a density; the subscript $i$ refers to the "channel" $i$. All the defined variables obey to balance equations which are due to the conservation of the total mass and of the total volume for each sub-system: the ball-mill, the sump and the hydrocyclone. However, the measured variables are generally inconsistent from the material balance viewpoint; but as we shall see in the next section the redundancy of the data are used to adjust them.

2 - THE PRINCIPLE OF STABILIZING A MASS-BALANCE

The real value $Q_i^*, d_i^* (i = 1, 2, 5, 6, 9, 11, 12)$ satisfy the balance equations expressed in terms of volumetric flow-rate and mass flow-rate. For each sub-system we have:

**ball-mill**

\[ f_4 = \frac{Q_1^*}{d_1^*} + Q_2^* + Q_{11}^* - Q_5^* = 0 \]  \hspace{1cm} (1)

\[ g_4 = Q_1^* + Q_2^* + Q_{11}^* d_{11}^* - Q_5^* d_5^* = 0 \]

**sump**

\[ f_8 = Q_5^* + Q_6^* - Q_9^* = 0 \]  \hspace{1cm} (2)

\[ g_8 = Q_5^* d_5^* + Q_6^* d_6^* - Q_9^* d_9^* = 0 \]

**hydrocyclone**

\[ f_{10} = Q_9^* - Q_{11}^* - Q_{12}^* = 0 \]  \hspace{1cm} (3)

\[ g_{10} = Q_9^* d_9^* - Q_{11}^* d_{11}^* - Q_{12}^* d_{12}^* = 0 \]

(In our case, $d_1^*$ is a constant factor which represents the mineral density; therefore $d_1$ has not to
be estimated). Practice shown that measurements $Q_i$, $d_i$ carried out on the process only partially verify these balance equations; the reason is that the measurements do not present an accurate picture of the real magnitudes.

A certain number of error factors are brought into evidence, from which we may isolate the following:

- random factors,
- accidental factors,
- systematic factors.

In the following sections we focus our attention only on random factors and we suppose that all the measured variables are realizations of random variables.

In term of maximum probability, the best estimator $\hat{Q}$, $\hat{d}$ of the real magnitudes $Q^*$, $d^*$ is that which maximize the probability density of the observed values $Q$, $d$ subject to the mass balance constraints.

With a normal probability density and provided the variance $p_{Qi}$ and $p_{di}$ of the $Q$ and $d$ measurements are known, some classical transformations reduce this problem to the following:

$$\min \phi = \frac{1}{2} \sum_{i \in I} (\hat{Q}_i - Q_i)^2 p_{Qi}^{-1} + \sum_{j \in J} (\hat{d}_j - d_j)^2 p_{di}^{-1}$$

(4)

with $I = \{1, 2, 5, 6, 9, 11, 12\}$

$J = \{5, 9, 11, 12\}$

under the system of constraints (1, 2, 3) written in term of the estimates $\hat{Q}$, $\hat{d}$ instead of the real values $Q^*$, $d^*$.

The constraints (1, 2, 3) are attached to (4) by Lagrange multipliers $\lambda$ and $\mu$ which define the new functional:
The conditions of first order stationarity for $\mathcal{F}$ are:

\begin{align*}
\mathcal{F} &= \phi(Q_1, d_1) + \sum_{1 \in L} \lambda_1 f_1 + \sum_{1 \in L} \mu_1 g_1 \tag{5} \\
L &= \{4, 8, 10\}
\end{align*}

\begin{align*}
-p_{Q1}^{-1} (\hat{Q}_1 - Q_1) + \lambda_4 d_1 + \mu_4 &= 0 \\
-p_{Q2}^{-1} (\hat{Q}_2 - Q_2) + \lambda_4 + \mu_4 &= 0 \\
-p_{Q11}^{-1} (\hat{Q}_{11} - Q_{11}) + \lambda_4 + \mu_4 \hat{d}_{11} - \lambda_{10} - \mu_{10} \hat{d}_{11} &= 0 \tag{6-1} \\
-p_{Q5}^{-1} (\hat{Q}_5 - Q_5) - \lambda_4 - \mu_4 \hat{d}_5 + \lambda_8 + \mu_8 \hat{d}_5 &= 0 \\
\frac{\hat{Q}_1}{d_1} + \frac{\hat{Q}_2}{d_2} + \frac{\hat{Q}_{11}}{d_{11}} - \frac{\hat{Q}_5}{d_5} &= 0 \\
p_{d11}^{-1} (\hat{d}_{11} - d_{11}) + \mu_4 \hat{Q}_{11} - \mu_{10} \hat{Q}_{11} &= 0 \\
p_{d5}^{-1} (\hat{d}_5 - d_5) + \mu_4 \hat{Q}_5 + \mu_8 \hat{Q}_5 &= 0 \tag{6-2} \\
\frac{\hat{Q}_1}{d_1} + \frac{\hat{Q}_2}{d_2} + \frac{\hat{Q}_{11}}{d_{11}} - \frac{\hat{Q}_5}{d_5} &= 0 \\
p_{Q6}^{-1} (\hat{Q}_6 - Q_6) + \lambda_8 + \mu_8 &= 0 \\
p_{Q9}^{-1} (\hat{Q}_9 - Q_9) - \lambda_8 - \mu_8 \hat{d}_9 + \lambda_{10} + \mu_{10} \hat{d}_9 &= 0 \tag{7-1} \\
\hat{Q}_5 + \hat{Q}_6 - \hat{Q}_9 &= 0
\end{align*}
\[
\begin{align*}
-\frac{1}{p_d9} (\hat{d}_9 - d_9) - \mu_8 \hat{q}_9 + \mu_{10} \hat{q}_9 &= 0 \\
\hat{q}_9 \hat{d}_9 + \hat{q}_6 - \hat{q}_9 \hat{d}_9 &= 0 \\
-\frac{1}{p_d12} (\hat{d}_{12} - d_{12}) - \lambda_{10} - \mu_{10} \hat{d}_{12} &= 0 \\
\hat{q}_{12} - \hat{q}_{11} - \hat{q}_{12} &= 0 \\
-\frac{1}{p_d12} (\hat{d}_{12} - d_{12}) - \mu_{10} \hat{q}_{12} &= 0 \\
\hat{q}_9 \hat{d}_9 - \hat{q}_{11} \hat{d}_{11} - \hat{q}_{12} \hat{d}_{12} &= 0
\end{align*}
\]

In practice an analytical solution is not possible to express on account of the strong linking between these 17 equations. Classical methods can be used to solve them such as gradient or Newton algorithms, linearisation. Our purpose here is to reduce the apparent complexity of the system (6, 7, 8) by partitioning the criterion into sub-criterions each of them being easier to optimize than . The algorithm is then a decentralized one which use basic principles of decomposition and coordination.

The partitionning is done through a physical approach; the process can be naturally divided into three sub-systems: ball-mill, sump, hydrocyclone. These sub-systems and their connexions are summarized at figure 2.

By using the concept of decomposition - coordination the mass-balancing is first locally realised on each sub-system; then to take into account the linking between the sub-systems, a coordination parameter is adjust to transfer the local solution to the global solution.
Let us now illustrate this general principle on our problem.

Figure 2 - Decomposition of the process.

If $X$ is the aggregate vector of $Q$, $d$, $\lambda$, equations (6, 7, 8) can be written as:

$$X = F(X)$$

The foregoing equation shown that $X$ is the fixed point of the $F$ application; to obtain the $X$ solution we suggest to apply a sequential-parallel procedure (Jacobi-Gauss) which is divided in:

- A parallel algorithm: equations (6, 7, 8) are partitioning into three subset as previously defined.
- a sequential algorithm; for each preceding subset the variables Q and d are sequentially obtained with a two-steps relaxation calculus.

Sub-system 1: Ball-mill.

Assume that \( \lambda_8, \lambda_{10}, \mu_8, \mu_{10} \) are given numbers chosen by the coordination level; thus equation (6) can be solved in term of \( \tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_5, \tilde{Q}_{11}, \tilde{d}_5, \tilde{d}_{11}, \lambda_4, \mu_4 \).

Sub-system 1-1: Ball-mill flow-rate

Assume that \( \tilde{d}_1 \) are given numbers, then from equation (6-1) we found \( \tilde{Q}_i, i \in I_4 = \{1, 2, 5, 11\} \)

\[
\begin{align*}
\hat{Q}_1 &= \tilde{Q}_1 - p_{Q1} \lambda_4 / d_1 \\
\hat{Q}_2 &= \tilde{Q}_2 - p_{Q2} \lambda_4 \\
\hat{Q}_5 &= \tilde{Q}_5 + p_{Q5} \lambda_4 \\
\hat{Q}_{11} &= \tilde{Q}_{11} + p_{11} \lambda_4
\end{align*}
\]

with

\[
\lambda_4 = (\tilde{Q}_1 / d_1 + \tilde{Q}_2 + \tilde{Q}_{11} - \tilde{Q}_5) / (p_{Q1} / d_1^2 + p_{Q2} + p_{Q5} + p_{Q11})
\]

\[
\begin{align*}
\hat{Q}_1 &= Q_1 - p_{Q1} \mu_4 \\
\hat{Q}_2 &= Q_2 - p_{Q2} \mu_4 \\
\hat{Q}_5 &= Q_5 - p_{Q5} (\lambda_8 + \tilde{d}_5 (\mu_8 - \mu_4)) \\
\hat{Q}_{11} &= Q_{11} - p_{Q11} (-\lambda_{10} + \tilde{d}_{11} (\mu_4 - \mu_{10}))
\end{align*}
\]

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Sub-system 1-2: Ball-mill density.

Assuming that the \( \hat{Q} \) are given the solution of equation (6-2) is expressed \( d_j, j \in J_4 = \{5, 11\} \)

\[
\hat{d}_5 = \hat{d}_5 + p_{d5} \mu_4 \hat{Q}_5
\]

\[
\hat{d}_{11} = \hat{d}_{11} - p_{d11} \mu_4 \hat{Q}_{11}
\]

\[
\mu_4 = (\hat{Q}_1 + \hat{Q}_2 + \hat{Q}_{11} d_{11} - \hat{Q}_5 d_5)/ (\hat{Q}_{11}^2 p_{d11} + \hat{Q}_5^2 p_{d5})
\]

\[
\hat{d}_5 = d_5 - p_{d5} \mu_8 \hat{Q}_5
\]

\[
\hat{d}_{11} = d_{11} + p_{d11} \mu_{10} \hat{Q}_{11}
\]

To sum up, the resolving algorithm for this first sub-system is written as:

\[
\hat{X}_{k+1} = F(\hat{Y}^k)
\]

\[
\hat{Y}_{k+1} = G(\hat{X}^k+1)
\]

where \( X \) and \( Y \) are respectively the flow-rate and density vectors and where \( k \) denotes the \( k+1 \) iteration. This two-step relaxed algorithm is well converging in our case because the F and G applications are contractant.

Sub-system 2: Sump

Sub-system 3: Hydrocyclone

For the two sub-systems, a similar two-step relaxation algorithm is used to estimated the \( \hat{Q}, \hat{d} \) variables.
3 - EXPERIMENTAL RESULTS

The relaxed balancing equilibration have been programmed and entered as a real time task on a process computer connected through the sensors at the grinding-classification process.

As we have previously said, this program run only under steady state conditions for the process: thereby on line tests are undertaken on the circuit to determine the time period in which the variables are static. This point has been solved by using statistic tests, but their presentation is outside the presents purpose.

The whole procedure of data equilibration is resumed in figure 3.

The first level performs the collect of data while the second level rejects the undoubtelly untrue data. An eventual third level filters the data in order to give mean value.

The fourth level improves the robustness of the data; this is done locally for each sub-system by evaluating the residuals of the mass and volumetric balance equations; in case of small residual the procedure stops here.

The fifth and sixth levels are connected to the decentralized and relaxed balancing problem.

An eventual eighth level gives the possibility to take into account the mass-balance equilibration for each size of particles.

A typical result appears on figure 4 for the measurements and on figure 5 for the adjusted data.
CONCLUSION

An application of the decentralized calculation has been presented to a balance equilibration problem, based on noisy measurements of the system variables. The algorithm proposed takes into account standard informations collected on a grinding classification circuit: volumetric flow-rate, mass flow-rate, density.

Several extensions of the method are currently being set-up in view of simultaneously taking into account the effect of random errors and the effect of systematic errors such as sensor breakdown.

REFERENCE


M. ROESCH, J. RAGOT - Validation de mesures par filtrage et équilibrage de bilan. MECO'82 IASTED Symposium, 1-3 September 1982, Tunis, Tunisie.

Figure 3 - Hierarchical mass-balance equilibration
### Mill, Sump, Hydrocyclone

<table>
<thead>
<tr>
<th></th>
<th>Mill</th>
<th>Sump</th>
<th>Hydrocyclone</th>
</tr>
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<tr>
<td>$Q_2$</td>
<td>153,000</td>
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<td>153,000</td>
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<tr>
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**FIG. 4 - MEASUREMENTS**

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</tr>
</tbody>
</table>

**FIG. 5 - ESTIMATIONS**