PARTICLE COMPOSITION PREDICTION IN ORE GRINDING

GILES BARBERY ¹
DONALD LEROUX ¹

ABSTRACT

Grinding of ores before physical separation processes is only carried out to produce particles that are close to what researchers call "liberation". The paper describes a method that has been developed to predict the complete particle composition distribution after fragmentation of heterogeneous materials. The principle of the method is based on a modelling of ore texture, fragmentation, and the interaction between the two subprocesses. The models retained are stochastic in nature, and can be calibrated by image analysis or heavy liquid testing. A computer programme which incorporates the models is described. Examples are given for applications to coal washability curve, and to grinding of taconites.

¹ GRAIIM, Department of Mining and Metallurgy
LAVAL University, Quebec G1K 7P4, CANADA
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Gilles Barbery and Donald Leroux
GRAIIM, Department of Mining and Metallurgy
LAVAL University, Québec G1K 7P4, Canada

Abstract

Grinding of ores before physical separation processes is only carried out to produce particles that are close to what researchers call "liberation". The paper describes a method that has been developed to predict the complete particle composition distribution after fragmentation of heterogeneous materials. The principle of the method is based on a modelling of ore texture, fragmentation, and the interaction between the two subprocesses. The models retained are stochastic in nature, and can be calibrated by image analysis or heavy liquid testing. A computer programme which incorporates the models is described. Examples are given for applications to coal washability curve, and to grinding of taconites.

1. Introduction

In mineral processing, grinding is rarely used to reduce the size of particles; in processes involving mineral separation based on physical or physico-chemical methods, grinding is used as feed preparation in order to produce particles that are monominerallic
in their composition, a situation which is traditionally described as "liberation". Complete liberation is only attainable at very fine sizes, and is rarely justified for practical or economical reasons. Partial liberation is of importance in industrial applications, and it is a major particle population characteristic. In fact, since liberation is only a concept with limited applicability, it is better to reason in terms of particle composition distribution with respect to the various phases that are present. Measurement of this distribution is not easy, and various reviews have been made of the available methods (Barbery, 1985, Barbery, 1986). Due to the difficulty and cost of the measurements, a promising field of activity appears to be the development of prediction methods. The paper presents the principles of a model for particle composition prediction and its validation from published data, in which measurements were made by physical separation methods.

2. Principles of particle composition prediction simulators

Barbery and Leroux (1987a) have presented a review of the various methods that have been introduced, over the past fifty years, to predict liberation in ore grinding. The various researchers have progressively identified the following parts in any model for particle composition prediction:

- an ore texture descriptor, which should be amenable to calibration, and incorporate stochastic properties.

- a particle generation descriptor, again amenable to calibration
and containing stochastic properties. The function of this descriptor is to quantify the distribution of particles shapes and sizes obtained by grinding.

- a description of the interaction between the two previous processes of texture and particle generation.

All researchers active in the field assume that the two stochastic processes are unrelated. This an assumption is known as the "pure transgranular breakage" mechanism, which has strong theoretical limitations, but which can be modified, as will be seen in part 3 of the paper, to accommodate selective breakage of some phases. The major developments that have taken place over the past five years in this field relate to this assumption, and to the case of binary textures, in which one mineral of interest is quantified, all the other phases being grouped together.

The authors have developed models based upon theoretical derivations by Davy (1984). The models are presented in various papers (Barbery, 1986 and 1987, Barbery and Leroux, 1987a and b) and persons interested in the derivation should refer to these publications. The main features of the models are illustrated below:

- texture description is quantified by the covariance function of the random three dimensional process which has resulted in the texture. The covariance function is defined by:

\[ C(r) = E[(f(x)-p_1)\ (f(x+r)-p_1)] \]
In this expression \( f(x) \) is the indicator function of the material (assumed to be binary for simplicity of presentation), being equal to 1 when point \( x \) is in phase 1, the mathematical expectation is taken for all pair of points at a distance \( r \) and for all directions. \( p_1 \) is the volume fraction of phase 1 in the ore and \( p_1 = E[f(x)] \). \( C(r) \) can be measured with image analyzers. It has the following properties: \( C(0) = p_1(1-p_1), C(\infty) = 0, C'(0) = -So_1/4 \) where \( So_1 \) is the specific area of contact between phases 0 and 1 per unit volume of ore. For modelling purposes, it is convenient to reduce the number of parameters to be quantified and the following texture models have been tested: Poisson polyhedra (Barbery, 1987), Boolean process with Poisson polyhedra as primary grains (Barbery and Leroux, 1987a and b). Figure 1 is an example of the apparent complex texture which results from the second textural model.

- Particle generation is described by the probability that two points separated by a distance 1 in the ore are present in the same particle after grinding, \( P(1) \). It is related to the distribution of random intercepts length across particles, \( f(1) \), which can be measured by image analysis of particles in the ground ore mounted randomly. It can also be derived for simple geometrical shapes, such as spheres. King (1984) has carried out measurements of \( f(1) \) on actual particles produced during the grinding of an iron ore. His measured function, as well as that derived for spheres, is used in the models. Particles are assumed to be convex in shape.

- Integration of texture and breakage is assumed to be an
independent process resulting in pure transgranular breakage. Barbery and Leroux (1987b) have outlined the major limitations of this assumption, and put forwards a modification of the models so that subtextural structures are considered, each having its own \( P(l) \) leading to selective breakage of each texture. An example of the application of such a model is given in part 3 of the present paper, and is completely described in Barbery and Leroux (1987c).

The mathematical solution of the problem involves the application of integral and stochastic geometry, and is given in Davy (1984) and Barbery and Leroux (1987a). The method provides the solution for a close size fraction of the ground ore, in which particles are assumed to have the statistically same volume \( V \). The particle composition distribution is quantified by the grade probability distribution, \( g(m) \), where \( m = \frac{V_0}{V} \), \( 0 \) being the index characterizing phase \( 0 \). It is possible to estimate the first two moments of \( g(m) \) from the origin by the following equations:

\[
\begin{align*}
m_1 &= p_0 \int_{0}^{\infty} l \, P(l) \left[ p_0 (1-p_0) - C(l) \right] \, dl \\
m_2 &= p_0 - \frac{\int_{0}^{\infty} l^2 P(l) \, dl}{\int_{0}^{\infty} l P(l) \, dl}
\end{align*}
\]

Obviously the independance of breakage and texture is reflected in the constancy of the first moment with respect to size. The limiting value for large particle size gives \( m_2 = p_0^2 \) indicating that all particles have the same composition, and for very small sizes, \( m_2 = p_0 \), a characteristic of completely liberated binary particles, for which the variance of \( g(m) \) is \( p_0 (1-p_0) \).
An incomplete Beta function can be fitted to the interval \([0,1]\), on which \(g(m)\) is valid, for the two moments \(m_1\) and \(m_2\). From \(g(m)\), separability curves and washability curves can be drawn, such as the one presented on Figure 2. A computer programme, called MODLIB has been prepared to demonstrate the various models and calculate washability curves. It enables to test the following combinations:

- **Ore textures**: Poisson polyhedra
  
  Boolean process with Poisson polyhedra as primary grains (phase \(0 = \) grains or phase \(0 = \) complement)

- **Particles**: monodisperse spheres
  
  Particles characterized by King (1984)

The use of this programme enables to assess variations in particle composition distribution as a function of size and texture. Figure 3 gives the variation in the relative variance of \(g(m)\), \(RV\), where

\[
RV = \frac{(m_2 - m_1^2)}{(p_0 - p_0^2)^{-1}}
\]

as a function of the relative particle size. \(D_{80}\) for the Boolean scheme represents the diameter of a sphere in which 80\% by volume of the primary grains would fit, and can be considered as a size scale for texture. For low grade materials, which can be assumed to have a Poisson polyhedra texture with low volume fraction, \(C(l) = 1 - \exp \left( -\pi Asl \right)\), and

\[
RV = 1 - \frac{\int_0^{D_{80}} P(1) \exp \left( -\pi Asl \right) dl}{\int_0^{D_{80}} P(1) dl}
\]
Which makes RV independant of grade, and only dependant upon the
relative scale of particles/grains, and on particle shape. For
very fine particles with respect to textural size, $C(l)$ can be
developed around zero to $C(l) = p_0(1-p_0) - \frac{S_0}{4}$, and RV becomes:

$$RV = \frac{\int_0^{\infty} l^3 P(l) dl}{4 \int_0^{\infty} l^2 P(l) dl}$$

which is only dependant upon particle shape, size and specific
surface area of interphase in the ore, in other words, independant
of grade and textural type, as can be deducted from some of Meloy
(1985) theorems.

Such variations of $g(m)$ with size can be used in the
reconstruction of the overall particle composition distribution.
For modelling purposes, various models exist for the prediction of
particle size distribution in unit operations such as ball milling
or rod milling (see for example Austin et al, 1984). Overall $G(m)$
can be obtained from:

$$G(m) = \sum p(i)g(m,i)$$

where $p(i)$ is the fractional volume contained in size range $i$, for
which the size should be taken as $(didi+1)^{0.5}$. Again complete
washability curves can be drawn for the overall size distribution,
based upon $g(m,i)$ functions, specific gravity of the phases, and
mass fractions in various size ranges. This analysis would be
completely applicable in the case of ores in which mineral
constituents have similar specific gravity. In the case of major differences between specific gravity and for closed circuit grinding with a classifier in which particles behave differentially depending upon their s.g. as well as their size, the method should be modified to take into account the effect high s.g. particles returning to the grinding mill. This problem has not received a solution at the present time.

3. Examples of applications

There are at present only limited examples on which it is possible to apply and test the models presented above, mainly due to the difficulties encountered either in quantifying texture, or in measuring particle composition distribution. Only two examples will be given here, one for iron ore grinding, and the other for coal washability curves prediction.

3.1. Taconite grinding

The example is taken from the very detailed analysis of magnetic iron ore grinding carried out by Wiegel (1976). Screened fractions of magnetic taconites ground at various sizes were analyzed for their magnetite content by the Davis tube method. In this case, transgranular breakage could be accepted since grade was almost independent of size. A model consisting of a single Boolean-Poisson texture with King fragment shape (BOOKING) was tested, and the results are given on Figure 4, in which a comparison is made of Wiegel's predictions, BOOKING predictions and experimental results. The agreement is very good for all size
3.2. Model calibration with coal washability data

Since the models enable to predict washability curves, it has been decided to study the possibility of calibrating model parameters with actual washability curves. There is extensive data available on coal washability, since this is a characterization procedure used extensively in coal washing or in the assessment of coal samples. In this example, the data of Wizzard et al. (1983) has been used, which give complete washability information on a size by size basis for size range from 100 mm to 1 mm. In opposition to the case of taconite, coal breakage cannot be assumed to be purely transgranular, since, for example, ash content varies markedly with size. For this case, and in order to maintain the great simplicity of transgranular breakage, the following modification is introduced, with reference to Figure 5. The material is assumed to consist of two subtextures, one containing two phases, and the other, one. In the case of coal for example subtexture 1 would be pure "shale", and subtexture 2 will be a coal "seam" containing carbonaceous matter and ash forming minerals (shale). The two subtextures would break independently, in subtexture 2, the two phases would be submitted to pure transgranular breakage. This model introduces, in addition to the parameters for quantification of subtexture 2, the volumetric proportion of "seam" with respect to the overall material.

An additional difficulty is also encountered in the case of coal washability curve analysis. "Coal" is not a mineral per se, and
its specific gravity is not constant. In order to transform washability information relating \( s.g. \) to mass and ash distribution, into volumetric data, the specific gravity of each constituent must be known. For shale, there is no great difficulty in assuming that the specific gravity has a single value. For "coal", a distribution must be included, which makes the analysis more complex. As it can be seen in Figure 6, a distribution of specific gravity values is transferred from the various particle compositions to the actual washability curves. In the present work, it has been found convenient to use a generalized exponential distribution for coal \( s.g. \), with two unknown values \( \bar{s}g \) and \( s_g^{\min} \). Shale \( s.g. \) was taken at 2.8. The application of a BOOKING model to "seam" breakage and multiple regression analysis led to the following parameters: for BOOKING \( ds_0 \) 25000 micrometers and volumetric shale content of 0.037, the overall "seam" proportion was .806 by mass, and "coal" \( s.g. \) distribution had a \( s_g \) of 1.328 and a \( s_g^{\min} \) of 1.267. The resulting size distribution of run of mine material, "seam" and "shale" is given in Figure 7. The adequacy in the washability curve reconstruction can be assessed in Figure 8, for four typical size fractions.
4. Discussion

The results presented in the two examples given in the preceding section demonstrate that the models based on a fundamental analysis of heterogeneous solids breakage, are able to represent well existing experimental data. The apparent limitations of the model, which are mainly related to the assumption concerning independance of fracture and texture, are partially removed by assuming a polytextural description of the heterogeneous solid, and allowing selective fragmentation of each subtexture. For coal breakage, for example, and for the purpose of describing ash distribution in broken coal, a textural description with two subtextures is shown to be sufficient with the data that was used. For a complete description of coal breakage, including the behaviour of pyrite sulphur, it would be necessary to incorporate another textural descriptor for this phase.

It should be pointed out however, that the results do not completely validate the model, in particular from a lack of direct measurement of textural parameters in the unbroken material. Barbery and Leroux (1987b) have shown that in the case of pyrite in gold ores, the quantification of texture by intercept length distribution allowed to predict, according to the BOOKING model, liberation in ground material, and that the results were in close agreement with experimental results. For the two examples given in the present paper, no information could be obtained on unbroken material, so that the information derived is more a calibration than a verification of the model assumptions. Direct measurements of ore texture are very
important, and should be done in order to verify the model. Present day image analyzers, as reviewed by Barbery (1985) are able to provide the quantitative description of texture, and should be used. The present authors have installed, in February 1987, an image analysis system based on a scanning electron microscope, on which they will carry out such studies. For practical applications, in particular for the integration of particle composition distribution models in simulation models for mineral processing circuits, it is felt that the approach retained for the examples of the present paper is sufficient, since it does not require major modifications to experimental procedures used in laboratories or plants.

5. Conclusions

A model has been derived from a fundamental analysis of the breakage of heterogeneous materials, which includes modelling of texture and of particle production. It assumes pure transgranular breakage. The results of the model enable to predict particle composition distribution as a function of size, and to calculate washability or separability curves. The model can be used directly if grade of broken ore does not vary with size. It is shown that it can be modified to cope with selective breakage. An example of an application is given for coal breakage, for which textural model calibration can be obtained from conventional washability curves. Lack of quantitative information on texture of heterogeneous solids is a serious present limitation in model validation, but its calibration for practical applications can be done using the methods presented.
REFERENCES


Barbery, G. and Leroux, D., 1987c, Calibration of textural models for liberation prediction using heavy liquid separation results, paper presented at AIME Annual Meeting, Denver.


Figure 1. Example of a Boolean texture with Poisson polyhedra as primary grains. Plane section through a simulated ore.

Figure 2. Example of washability curves obtained by particle composition prediction after breakage of a Poisson polyhedra texture ($p_0=0.1$, s.g. of phase 0:4.5, s.g. of phase 1:2.8) for spherical particles having a diameter equal to the dso of the texture. (1) mass fraction in floats/s.g. (2) mass fraction in floats/floats grade (3) distribution in floats/floats grade.
Figure 3. Variation in relative variance of particle composition distribution for BOOKING model: example of variation with \( \frac{d}{d_{50}} \), for two volumetric concentrations. Boolean texture with primary Poisson polyhedra grains, particles as measured by King (1984).

Figure 4. Comparison of Wiegel (1978) experimental results with his model prediction (□) and those of BOOKING (○). Figures indicate volumetric fraction going to magnetic concentrate. Points on the right hand side represent coarse particle sizes.
Figure 5. Example of a bitextural material: "seam" and "shale" for coal.

Figure 6. Effect of a distribution in specific gravity of a phase on heavy liquid separation.
Figure 7. Simulation of coal washability results for 4 size fractions. D in micrometers, (●) experimental results, — simulation.

Figure 8. Size distribution of run-of-mine material (experimental values ○), "coal" (▲) and "shale" according to the model calibration(○).