A Simple Model for Entrainment in Conventional Flotation Cells

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ABSTRACT

Entrainment, a well known process occurring in any flotation cell, can be defined as the undesirable and unpreventable classification of ore particles based exclusively upon size and density which happens in varying degree simultaneously to the true flotation process. Being independent of particle surface properties, entrainment can be highly detrimental to the selectivity of flotation operations. The extent of the entrainment process is usually associated with the recovery of feed water to the cell concentrate which accounts for the common plant practice of keeping the feed water recovery to a minimum. Throughout this work, entrainment has been divided into three sub-processes by which (i) some of the low inertia particles present in the pulp region of the cell are caught by strong water streamlines and are carried away from the impeller boundaries, (ii) as a consequence, a suspension of these particles is generated in the top levels of the pulp region and (iii) small amounts of this suspension are continuously pushed up into the froth region by ascending air bubbles. This approach, plus the assumption that entrained particles drain from the froth at the same rate as that of water, allow the development of a model for the entrainment process which contains only a single empirical parameter. Such a parameter can be estimated by fitting down-the-bank data from the fine size intervals of the non-floatable gangue. Conversion to the valuable mineral is based simply upon the density ratio. Published laboratorial data as well as industrial data from the Kambalda nickel circuit (Australia) were used for the preliminary validation of the model.

KEY WORDS: 1: Flotation, 2: Entrainment, 3: Modelling, 4: Simulation.

1. INTRODUCTION

There are at least two well known processes by which ore particles report to the concentrate of a conventional flotation cell; namely true flotation and entrainment.

In the true flotation process, ore particles adhere to gas bubbles due to either natural or induced surface hydrophobicity. Efficient true flotation is dependent upon the success of three sub-processes: (i) bubble particle collision, (ii) bubble hydrophobic particle attachment and (iii)
transport of the bubble-particle aggregates to the concentrate launder against detachment forces [1,2,3]. Process selectivity is achieved via adjustment of pulp chemical environment (collector type and concentration, activators, depressants, pH, Eh, etc) in order to ensure an appropriate difference between the hydrophobicity of valuable and non-valuable particles. A varying amount of gangue is always recovered locked in composite particles as, due to economical considerations, complete liberation is not usually attempted.

In the entrainment process, low inertia particles simply follow the water transfer to either concentrate or tailing stream irrespective of any surface property. Despite the fact that some fine valuable particles can be recovered by entrainment, the process is always a drawback for two major reasons: (a) it allows the recovery of both liberated gangue and low grade composite particles with consequent decrease in the concentrate grade; (b) under standard plant operation most of the entrained particles (valuable and gangue) are carried along to the tailings according to the usual water split with consequent decrease in the recovery of valuables. Entrainment is an even more serious problem when processing low grade fine disseminated ores.

The entrainment process has been extensively investigated and a linear relationship between water recovery and solids entrainment recovery, as presented in Figure 1 [4] has been verified by several authors [5,6,7,8,9]. It has been shown that such relationship holds above a minimum feed water recovery [8,9]. The influence of pulp density is highlighted as well in Figure 1. It is generally accepted that the extent of entrainment can be minimised via the control of operational factors which affect feed water recovery, such as air flowrate, overall bubble surface area and froth drainage characteristics [4,9,10,11,12].

![Figure 1 - The cleaner at Mount Lyell, showing the effect of pulp dilution on the gangue/water ratio in the concentrate [1].](image)
Figure 2 - Effect of particle size on the degree of entrainment for hydrophobic silica.

Table I - Effect of particle density on the degree of entrainment in a Hallimond tube. For each solid $D_{50}$ represents the particle size which presents a 50% chance of entrainment.

<table>
<thead>
<tr>
<th>Solid</th>
<th>Density (g/cm$^3$)</th>
<th>$D_{50}$ (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe(Ni)</td>
<td>8.00</td>
<td>22</td>
</tr>
<tr>
<td>NiS</td>
<td>5.40</td>
<td>50</td>
</tr>
<tr>
<td>Fe$_3$O$_4$</td>
<td>5.20</td>
<td>50</td>
</tr>
<tr>
<td>ZnS</td>
<td>4.00</td>
<td>76</td>
</tr>
<tr>
<td>MgCO$_3$</td>
<td>2.94</td>
<td>120</td>
</tr>
<tr>
<td>mica</td>
<td>2.93</td>
<td>245</td>
</tr>
<tr>
<td>CaCO$_3$</td>
<td>2.71</td>
<td>145</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>2.65</td>
<td>134</td>
</tr>
<tr>
<td>bones</td>
<td>2.17</td>
<td>178</td>
</tr>
<tr>
<td>NaCl</td>
<td>2.17</td>
<td>21</td>
</tr>
</tbody>
</table>
Under correct cell operation, a quiescent zone, i.e., a zone with relative low turbulence, must exist underneath the froth region. The main reasons are both to prevent froth shaking, which would otherwise lead to decreasing attached particle recovery, and to keep water streamlines from crossing the pulp-froth interface, which would bring along a load of non-entrained particles. According to the latter, under proper operational conditions, water may only enter the froth region via an indirect mechanism. In fact water recovery tends to increase with increasing overall bubble surface area, i.e., increasing air flow rate and decreasing individual bubble size.

The froth region can be seen as a barrier which physically separates the valuable mineral particles attached to the ascending bubbles from those in the gangue-rich suspension existing underneath the pulp-froth interface. However, as small amounts of this suspension are continuously introduced in the froth base, such a barrier is not impervious and entrained particles are always present in the froth region suspended in the water between the bubbles. Drainage of water, and consequently of entrained particles, occurs throughout the froth. Moys [13] investigating a continuously operated froth, observed a decrease in gangue content from about 80% to 40% right above the pulp-froth interface (first 5 cm) and from about 40% to 5% over the upper 25 cm. It is considered that a froth with characteristics which favor drainage (such as high depth with large bubbles and loose structure) is advantageous for grades. On the other hand, owing to lower drainage, a closely knit froth tends to increase recovery of both attached and entrained particles [14]. Drainage of the inter-bubble suspension can be improved by creating a negative water bias via the introduction of washing water on the top of the froth region. Washing water has a strong effect on feed water rejection which is recognized as one of the major advantages of industrial column flotation [12].

The effect of particle size, as presented in Figure 2 [6,15,16], has been well established in the literature, with recovery by entrainment increasing with decreasing particle size. The effect particle density is presented in table 1 [17].

Subrahmanyan and Forssberg [14] stressed the non-selectivity of entrainment in opposition to true flotation and referred to the work of Engelberg and Woodburn [9] who observed a linear relationship between mineral and water recovery in the case of fine hydrophobic pyrite and fine hydrophobic silica. Under the same conditions coarse pyrite recovery was found to be dependent upon hydrophobicity.

2. THE CAUSES OF THE ENTRAINMENT PROCESS

Smith and Warren, in a classical paper [7], recognized that the most common mechanisms proposed to explain entrainment are:
a) Water is dragged along by bubble walls, i.e., water is carried upwards in the bubble's hydrodynamic layer [18].

b) Water is transported in the wake of an ascending bubble [19].

Via relatively simple and sound calculations, the same authors demonstrated that none of these mechanisms alone can account for the water content usually verified in flotation froths. In this sense a third mechanism was suggested to be dominant:

c) Water is mechanically pushed up into the froth region by ascending swarms of bubbles.

The third mechanism leads to the simple assumption that at the bottom of the froth region the concentration of entrained particles in the inter bubble suspension is the same as that on the top of the pulp region.

In the present work, as discussed in more detail throughout the following sections, entrainment is divided into three sub-processes occurring in succession within the collection and quiescent zones as well as in the froth region of a conventional flotation cell. It is suggested that the primary cause of entrainment is the increasing tendency that low inertia particles present of following the water streamlines under the highly turbulent conditions prevailing in the pulp region. An assumption is made that under a limited impeller velocity, heavy particles tend to concentrate in the lower levels of the cell, whereas light particles are homogeneously spread all over the cell pulp volume. Therefore light particles would present a higher relative concentration underneath the pulp-froth interface. Woodburn et al. [20] carrying out tracer tests in a continuously operated flotation cell under "no flotation conditions", i.e., without collector and froth addition, showed that the relative residence time inside the cell increased with decreasing particle size.

3. QUALITATIVE DESCRIPTION OF THE ENTRAINMENT PROCESS

3.1 Entrainment in the Collection Zone

The impeller in a conventional flotation cell has a dual function, i.e., to suspend ore particle as well as to supply air bubbles with enough energy so that bubble-particle collision can occur. However, under the usual operational conditions of a conventional cell, owing to the flow pattern that has to be generated in order to suspend the pulp, many of the low inertia particles are caught by strong water streamlines and are withdrawn from the collection zone before even having a chance of collision with air bubbles. Along these streamlines the chances of collision
are quite small as, in this case, both bubbles and particles tend to move on the same direction with similar velocity. Collision is most likely to occur for particles whose trajectories intercept the high energy bubbles within the impeller boundaries.

The extent of entrainment increases with decreasing particle inertia. Therefore fine particles tend to be suspended homogeneously all over the pulp volume while coarse particles tend to accumulate on the lower levels of the cell closer to the impeller. That would account for the generally known difficulties regarding selective recovery of fines in conventional flotation. If in on hand suspended particles of colloidal dimensions are not likely to be recovered by true flotation even when presenting total hydrophobicity, on the other hand their recovery by entrainment can be quite confidently assumed as equal to the feed water recovery.

It is interesting to note that, within a given size distribution, the fine size intervals are generally the most liberated ones and thus should present the best separability between the valuable mineral and gangue. As long as these particles tend to follow the water partition curve, the restriction imposed by entrainment is detrimental not only to concentrate grade but as well to concentrate recovery once most of the water flow is directed to the tailings.

3.2. Entrainment in the Quiescent Zone

As a consequence of the flow pattern in the pulp region of a conventional cell, a dynamic suspension of low inertia particles is generated in the pulp top levels. Ascending gas bubbles cross this suspension and carry along water and entrained particles into the froth region. In this sense the concentration of entrained particles in the bottom layers of the froth should be very close to that in the quiescent zone suspension once no significant size classification is involved in this sub-process (predominant mechanism C, section 2). Many of the bubbles collapse on reaching the pulp froth interface releasing their load of attached and entrained particles back to the pulp region.

Therefore the flowrate of entrained particles of a given size interval into the froth region is directly related to both the overall gas surface effectively entering this region and to the concentration (mass of suspended particles / unit water volume) of such particles in the quiescent zone. This approach is in perfect concordance with the industrial data presented in Figure 1, as far as increasing overall gas surface leads to increasing feed water recovery; whereas increasing pulp density leads to increasing particle concentration in the quiescent zone.

3.3. Entrainment in the Froth Region
As soon as a bubble enters the froth region the drainage of its surrounding suspension starts. At the same time, due to the continuous arrival of bubbles at the pulp-froth interface, the froth region presents a net upward velocity. The amount of entrained particles that ultimately report to the cell concentrate is thus determined by the balance of two opposing tendencies: a) the upward velocity of inter bubble water which is given by the ascending velocity of bubbles across the froth and b) the downward drainage of feed water.

Two modes of drainage in the froth region are described by Cutting [21]:

a) Film water drainage - in which water and solids drain around the air bubbles. This mechanism is present in the whole froth structure, and proceeds at low rate over the whole froth volume.

b) Column drainage - in which material descends rapidly at single vertical locations in the froth. This can occur anywhere in plan location and starts at any vertical position where local concentration of solids inverts the hydrostatic gradient in the froth. Such a condition is unstable and, in a fluid froth, results in local collapse and consequent downward movement. The accumulation of the solids below aggravates the instability, creating an avalanche effect, which results in a rapid movement of material from the point of nucleation down into the pulp sweeping all in its path. The columns are limited usually to about 1 cm² in cross-sectional area.

Experimental results presented in the same work [21] indicate a decrease in water content from about 25% to 3% of froth volume across a froth height of 20 cm approximately.

4. QUANTITATIVE DESCRIPTION OF THE ENTRAINMENT PROCESS

4.1. Mass Transfer Fractions

In order to quantitatively describe the mass transfer involved in the entrainment processing of particles of a given mineral the following adimensional are proposed:

\[ E_{Fin} = \text{Entrainment fraction of size } i \text{ in the } n^{th} \text{ cell of a flotation bank. This adimensional represents the fraction of particles of size } i \text{ of a given mineral which are processed by entrainment in the } n^{th} \text{ cell (including both pulp and froth effects). The flowrate of entrained particles in the concentrate is given by: } CE_n = E_{Fin} \left(W_n - W_{n+1}\right) / W_n, \text{ where } W_n \text{ represents the flowrate of water in the feed to the } n^{th} \text{ cell. On the other hand, the flowrate of entrained particles to the tailings is given by: } TE_n = E_{Fin} W_{n+1} / W_n. \text{ Under usual plant operation } W_{n+1} \text{ is close to } W_n \text{ which implies that } TE_n \gg CE_n. \]
$Sfi$ = Streamline fraction of size interval $i$. This adimensional indicates the extent of entrainment in the pulp among particles of size interval $i$. It is given by the ratio of the concentration of particles of size $i$ in the quiescent zone to that in the feed to the cell. This fraction is expected to be an exclusive function of particle size, mineral density and flow pattern in the pulp region. No influence of hydrophobicity is considered. It is not supposed to be cell dependent as the cells of a flotation bank usually operate under approximately the same impeller velocity and as differences in air flowrate are not likely to affect particle suspension capability.

$Dfin$ = Drainage fraction of size interval $i$ from the froth of the $n$th cell. This fraction is given by the ratio of the drainage of particles of size $i$ to that of water and allows for differences in particle settling velocities to be accounted for. The drainage fraction is considered to be exclusively dependent on particle size and density as well as froth characteristics.

$Wfn$ = Water split in the froth of the $n$th cell. It is given by the ratio of the flowrate of water drained from the froth to the flowrate of water in the concentrate of cell $n$.

### 4.2. Mass Balance Equations

Assuming that the only way liberated non-floatable gangue particles can report to the concentrate of conventional flotation cells is via entrainment, the down-the-bank mass balance for such particles is given by [22]:

\[
F_{i, n+1} = I - Efin \left( \frac{W_{n+1}}{W_n} \right)
\]  

(1)

where $F_{i, n+1}$ represents the flowrate of liberated non-floatable gangue particles of size interval $i$ in the feed to the $n$th cell.

According to the definitions in section 4.1, the mass balance across the froth gives [22]:

\[
Efin = Sfi \left[ I \cdot Wfn(I - Dfin) \right]
\]  

(2)
4.3 Parameterisation of the Entrainment Process

As it was said entrainment in the collection zone is considered to be a process of classification based upon size and density. However, as a sharp classification is not likely, this process was modelled via a function of the hyperbolic secant form as usual for incomplete classification processes:

\[ S_f = \frac{2}{\exp(+.1517x_i/\xi) + \exp(-.1517x_i/\xi)} \]  

(3)

where \( x_i \) represents the nominal particle diameter of size interval \( i \) and \( \xi \) is the entrainment parameter. The constant values in equation (3) were set up in order that \( \xi \) would represent the size interval at which 50% of the particles are processed by entrainment. It must be emphasised that the entrainment parameter is independent of particle size. The greater the value of \( \xi \) the higher the magnitude of entrainment in the pulp region. It is important to note that, when considering particles of similar density in different size intervals, the probability of entrainment in the pulp, i.e. \( S_f \), increases with decreasing particle size. However, among particles of similar density in the same size interval, whether a given particle will be entrained or not is exclusively dependent upon the initial coordinates of the particle in the flow pattern generated in the pulp region of the cell.

As stated in section 3.2, there is no significant mechanism of classification involved in the entrainment of suspended particles in the quiescent zone. Therefore no parameterisation is necessary for this sub-process.

For the mathematical description of the drainage of entrained particles in the froth region, considering the low residence time of water in the froth region \([13,21]\), an assumption is made that the downward velocity of drained water is significantly higher than the settling velocities of the entrained particles. According to this approach, water and entrained particles would drain approximately at the same rate, i.e.:

\[ D_{fa} = 1 \]  

(4)

In this sense there is no significant size classification of entrained particles occurring in the froth and according to equation (2): \( E_{fa} = S_f \).
Figure 4 - Streamline fraction vs particle size as fitted in the Kambalda nickel circuit. In this case \( \zeta_{\text{range}} = 24.89 \, \mu\text{m} \).

Figure 5 - Cumulative recovery of silica down a rougher of the Kambalda nickel circuit. Results from simultaneous fitting of the entrainment parameter using two size intervals:
(a) size interval = 15.30 \( \mu\text{m} \).  
(b) size interval = 22.05 \( \mu\text{m} \).
5. ESTIMATION OF THE ENTRAINED PARAMETER USING FINE SIZE INTERVALS OF NON-FLOATABLE GAN GUE.

If liberation is approximately known and if there is available data on the gangue size-by-size recovery, it is possible to use the fine intervals of liberated gangue as tracers and fit the entrainment parameter so that the difference between observed and simulated gangue recovery values is iteratively set to a minimum. The numerical procedure relies on the minimisation of a SSQ function under the boundary conditions imposed by equation (1) and (2). Once the value of $\xi$ is estimated for the gangue, conversion to the valuable mineral is based upon the difference between specific densities according to the proposed equation [22]:

$$
\xi_{\text{valuable}} = \frac{\rho_{\text{gangue}}}{\rho_{\text{valuable}}} - \frac{\rho_{\text{pulp}}}{\rho_{\text{pulp}}} \cdot \xi_{\text{gangue}}
$$

In equation (5), $\xi_{\text{gangue}}$ is an indicator of the flow pattern of particles of a given density inside the flotation cell. The pulp density was included in order to account for particle buoyancy. Obviously heavier particles present higher inertia and are less subject to entrainment, which accounts for the inverse proportionality between $\xi_{\text{valuable}}$ and $(\rho_{\text{valuable}} - \rho_{\text{pulp}})$.

Figure 5 - Comparison between data published in the literature and results of simulation via equation (5).
6. PRELIMINARY MODEL VALIDATION

Data from table I was used in order to investigate Equation (5) as Drzymala's $D_{20}$ concept is very similar to the definition of $\xi$. Silica was arbitrarily chosen as the gangue and the pulp density in Drzymala's tests was assumed as that of pure water. The results are displayed in Figure 3. A straight line of unitary inclination represents Equation (5). Not all the points in Table I were plotted in Figure 3 as Equation (5) does not hold for either special particle shape or materials of low density nor soluble materials.

Down-the-bank industrial data from the Kambalda nickel circuit [23] was used for the investigation of equations (3) and (4) following the methodology described in section 5. Fitting of the entrainment parameter resulted in $\xi = 24.89 \mu m$. Further results are presented in Figures 4 and 5.

7. CONCLUSIONS

A simple model for the entrainment process based upon a single empirical parameter has been presented. The model structure is composed of two analytical and three empirical equations. Preliminary model validation has brought very positive results. One of the empirical equations agreed well with published data and the use of industrial data showed a case in which the model was able to fit eighteen experimental points from two different size intervals via its single empirical parameter.

However, the validation of one of the current model assumptions, namely that entrainment in conventional flotation cells is a classification process occurring primarily in the pulp region, is still to be corroborated. In this sense the investigation can be divided in two parts: a) carrying out tracer tests across the froth region and b) measurement of the degree of differential particle suspensibility in conventional flotation cells, i.e. differences in particle size concentration in various points of the pulp region. For the latter there are at least two appropriated procedures: b1) switching off the air flowrate and carrying out concentration measurements in an industrial flotation cell mixed under different impeller velocities; b2) operating the cell under normal conditions and using a device which is able to sample the entrainment particle suspension in the quiescent zone by collecting the suspended particles but not the ones attached to air bubbles. Data collection via the first methodology has already started by the JK centre's flotation group. A simple device which works according to the second methodology has been designed in this centre and performed exceptionally well under laboratory conditions.
It should be clear that, in order to incorporate the cases where classification in the froth is significant, the current model can be extended by simply connecting the drainage fraction to an empirical parameter via a size dependent relationship.

The present work constitutes a module of a comprehensive semi-empirical model for the industrial flotation process called Active Sites [22]. Such a model has been developed by first identifying and connecting the principal flotation sub-processes to a set of mass balance equations and then by describing each one of these sub-processes in as much detail as possible with the minimum number of empirical parameters.

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9. REFERENCES


