MONTE CARLO ANALYSIS OF THE FLOATABILITY COMPONENT APPROACH USED FOR FLOTATION MODELLING

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ABSTRACT

Floatability component modelling has been used within the literature to describe the flotation characteristics of the particles in the feed to a flotation process. Traditionally, the parameters associated with these models have been derived from a single flotation test of the feed stream.

Because of the inherent error associated with the experimental data used in parameter derivation, and the possible interaction of the different parameters in the model, the optimal values of the model parameters derived from experimental data cannot be trusted completely. A good model not only fits the data well but should contain stable parameters that do not change significantly when small error is introduced into the data set from which they are derived.

In this paper, the parameters derived from batch tests performed on multiple streams of an industrial flotation circuit are assessed using a Monte Carlo bootstrap technique. The analysis shows that the parameters from multiple batch flotation tests are unique and statistically stable - unlike the parameters derived from the traditional single batch flotation test. The findings are significant if floatability component models are to be used to predict flotation circuit performance with confidence.

INTRODUCTION

Developing a mathematical model of the flotation process has proved difficult because of the complex nature of the flotation system. The author, in a previous study (Runge et al, 1997), investigated the flotation behaviour of the different streams in an industrial flotation circuit. This study concluded that the propensity of the particles in the flotation feed stream to float was an important variable that must be included in the development of any flotation model. Floatability component modelling, used by Kelsall (1961), Imaizumi and Inoue (1965) and others, is a technique that can be used to represent this ore floatability.

Floatability component modelling utilises model parameters to lump factors that cannot be directly measured or calculated. These parameters are derived from experimental data from the process. Because of the inherent error associated with the experimental data used in parameter derivation and the possible interaction of the different parameters in the model, the optimal values of the model parameters produced using experimental data cannot be trusted completely. "These parameter values are the mean of the distribution of possible values." (Kocis, 1995)

A good model not only fits the data well but should contain stable parameters that do not change significantly when small error is introduced into the data set from which they are derived.

Traditionally, floatability component parameters have been derived from flotation feed laboratory float tests in which timed concentrate samples were collected. Harris (1997) states that parameter prediction can be improved by deriving the parameters of the floatability distribution from a number of tests performed on different streams in the flotation circuit. The increase in the size of the data set used to derive the parameters, decreases the size of the error associated with the flotation parameters.

The objective of this paper is to investigate the parameter stability of the floatability component modelling approach. This will be performed by analysing a set of industrial data and determining model parameters using a number of different strategies. A Monte Carlo bootstrap technique will be used to determine the stability of each set of derived flotation model parameters.

FLOATABILITY COMPONENT MODELLING

Bubble particle collection in flotation is generally considered to be a first order rate process.
The rate constant of this process \( (k) \) has been shown to be a function of the bubble surface area flux generated in the cell \( (S_b) \), the recovery across the froth phase \( (R_r) \) and the ore floatability \( (P) \) (Gorain et al, 1997).

\[
k = R_r S_b P
\]

Ore floatability is considered to be a property of the feed stream to the process and not dependent on the operating conditions within the flotation cell. Previous work by the author (Runge et al, 1997) and others (Imaizumi and Inoue, 1965) have shown that the ore floatability of a stream cannot be described by one number but by a distribution. Particles of a particular mineral within a flotation feed do not all react with the bubbles in the flotation process with the same rate. The flotation rate of a particle will depend on its particle size, reagent coverage, degree of liberation, surface composition, particle shape and its density. A flotation stream contains a large number of different particles and therefore should exhibit a distribution of flotation rates.

Floatability component modelling involves representing this distribution by a discrete number of floatability components, each with a unique ore floatability (Figure 1). The mass fraction of each component in the feed and the corresponding rate constant (under a specified set of cell operating conditions) are parameters of the distribution, derived using least squares minimisation techniques applied to experimental data collected from a process.

![Figure 1](image)

The classical two component model (Kelsall, 1961) is an example of the use of floatability components to represent the floatability distribution. This model has been used extensively by industry to characterise laboratory flotation data. Assuming a two component model, mineral recovery in a laboratory batch test is given by equation 2 and in a continuous flotation cell by equation 3.

\[
\text{Recovery} = m_1(1-\exp(-k_1 t)) + (1-m_1)(1-\exp(-k_2 t)) \quad (2)
\]

\[
\text{Recovery} = m_i \left( \frac{k_i C \tau}{1 + k_i C \tau} \right) + (1-m_i) \left( \frac{k_2 C \tau}{1 + k_2 C \tau} \right) \quad (3)
\]

Non-linear regression techniques are used to determine the optimum values of the floatability parameters \( (m_i, k_1, k_2) \). The flotation rate constant of the fast floating component in the feed, \( k_1 \) and \( k_2 \) - the flotation rate constants of the fast and slow components under the batch test operating conditions, \( C \) - the ratio between the flotation rate achieved in the continuous cell to that achieved in the batch test) which best match the experimental recovery data achieved in the batch laboratory tests and flotation circuit.

Most researchers have been unsuccessful at relating floatability component modelling to industrial scale behaviour. Harris (1997) states that this is due to error in the floatation parameter prediction. Because of the interaction of the different parameters in equation 2, experimental data from only one batch test are insufficient to estimate the optimum values of the floatability parameters. This problem can be overcome by performing laboratory flotation tests on a number of streams in a flotation circuit (Harris, 1997). Within the flotation units in a flotation circuit, there is separation of particles with different floatabilities into different streams - the fast floating particles concentrating into
the concentrate streams and slow floating particles concentrating into the tailing streams.

If one assumes that the ore floatability of particles does not change around the circuit (an assumption already validated for the data used in this paper (Runge et al., 1997)), the above regression analysis can be repeated on the data collected from the laboratory flotation tests on the different streams in the process. This regression can be constrained by the following assumptions:

- a floatability component will exhibit the same flotation rate in any stream in the process when floated using the same set of operating conditions;
- the total mass of a floatability component in the feed streams to a node in a flotation process will equal the total mass of that floatability component in the product streams of that same node.

By constraining the problem, the ratio between the number of parameters and the number of experimental data points is reduced. The error associated with the calculated parameters should, therefore, also be reduced.

The aim of this study is to determine the stability of the parameters of the floatability component model and how this stability is affected by the number of flotation tests used to determine the floatability components. It will also assess the effect of changing the number of components used in the analysis.

This was achieved using data collected from laboratory flotation tests performed on the different streams in an industrial circuit. All tests were performed using the same operating conditions. When the same operating conditions are used, the flotation rate of each component can be assumed to be the same in all tests.

**EXPERIMENTAL**

The experimental testwork for this study was undertaken within the lead cleaning circuit of Cominco's Red Dog operation on the 11th April, 1996. Figure 2 shows the Red Dog circuit configuration at the time of the testwork and denotes the streams investigated during the study.

The cleaning circuit consisted of a column unit and two conventional mechanical cells which acted as a cleaner scavenger. Feed to the cleaner circuit was the rougher combined concentrate. This stream had a P80 of approximately 20 microns and the major minerals present were galena, sphalerite and pyrite. For the purposes of this study, all the other gangue minerals present have been combined and will be referred to as non-sulphide gangue.

It should be noted that the Red Dog lead circuit flowsheet has been altered since this testwork was completed.

The test program consisted of collecting a sample of approximately 4.8 litres from each major stream in the cleaning circuit and transferring this sample to a Denver laboratory flotation cell. Plant water dosed with an appropriate amount of frother (MIBC) was used to make up the volume in the cell to 4.8 litres when necessary. After adjusting the air rate to 4.8 litres/min and impeller speed to 1300 rpm, timed concentrate samples were collected. Concentrate removal was performed at a constant rate of one stroke every 10 seconds using a paddle. The paddle was designed to ensure the depth of concentrate removal remained constant. Froth depth was maintained at a reasonably constant level throughout each experiment by the addition of frother dosed plant water based on visual observation.

![Figure 2 - Red Dog lead circuit denoting streams sampled for batch flotation tests](image)

- A - combined rougher cone; B - column fd; C - column cone; D - column tailing; E - chr scav cone; F - chr scav tailing

Each stream in the process was collected and floated sequentially. This procedure was adopted to minimise aging of samples during the time between collection and flotation. After all the major streams in the cleaning circuit had been floated, a sampling survey of the lead circuit was performed. This survey involved collecting a sub-sample of each major stream in the circuit at 10 minute intervals over a 40 minute period. Red Dog's on stream analysis system indicated that the circuit remained at steady state during the entire sampling campaign.
All samples collected during the flotation tests and sampling survey were weighed wet, filtered, dried and weighed again. Sub-samples were analysed by a combination of atomic absorption spectroscopy and titration methods to determine the lead, zinc and iron content of each sample. Mineral assay was estimated for each sample using the elemental assay and an assay to mineral conversion regime developed by Red Dog personnel.

This data set was used to calculate the mineral recovery in each stream of the circuit (based on the cleaning circuit feed) and the mineral recovery rate achieved by floating each of the six major streams in the Red Dog lead circuit in the batch tests. This data set has been previously published in Runge et al., 1997. The standard deviation of the batch test data was estimated using repeat batch tests. The standard deviation of the circuit mineral recovery was based on estimates of the standard deviation of the circuit assays and tonnage flows.

**MODEL PARAMETER ESTIMATION**

Using this data set, a number of different floatability component models representing the ore floatability of galena (the valuable mineral) in the Red Dog lead cleaning circuit were developed. These models had the following characteristics:

- A variation in the number of batch tests used to derive the parameters. Parameter stability is known to improve as the data set size used to derive the parameters is increased. These models were developed to investigate the change in parameter stability with a change in the number of batch laboratory flotation tests used in the analysis.

- A variation in the number of floatability components used to represent the floatability distribution. Parameter stability is also known to deteriorate as the number of parameters derived during fitting increases. These models were developed to investigate the impact of increasing parameterisation on the stability of the floatability component parameters.

Table 1 summarises the characteristics of the floatability component models developed for this analysis. During each regression, the parameters were derived such that the standard error of the system was minimised. Standard error (Equation 4) is defined as the difference between the experimental data ($d_{exp}$) and the prediction of this data set ($d_{pred}$) based on the model parameters. This difference is weighted by the standard deviation of each data point (SD) and the degrees of freedom associated with the analysis (i.e. number of data points (n) minus the number of parameters (p) in the regression).

$$\text{Standard Error} = \sqrt{\frac{\sum_{i=1}^{n} (d_{exp,i} - d_{pred,i})^2}{n - p}} \tag{4}$$

The minimum standard error associated with each of the floatability component models increased as the number of batch tests used in the calculation increased (Figure 3). The regression is able to match the experimental data better when fewer data points are used for model development.

There is also a significant improvement in the model fit when more than one floatability component is used in the model (Figure 3). This reflects the fact that ore floatability is a distributed property and one component is insufficient to represent this distribution adequately.

**Table 1 – Characteristics of the floatability component models**

<table>
<thead>
<tr>
<th>Batch Tests</th>
<th>No. Of Exp. Data Points</th>
<th>No. Of Comp. In Model</th>
<th>Parameters*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>A, C</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>A, C, F</td>
<td>23</td>
<td>2</td>
</tr>
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<td>6</td>
<td>A, B, C</td>
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</tr>
<tr>
<td>9</td>
<td>A, B, C</td>
<td>42</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>A, B, C</td>
<td>42</td>
<td>3</td>
</tr>
</tbody>
</table>

* $C_{Column}$ and $C_{Char Scav}$ are parameters introduced to maintain conservation of mass in the floatability components around the circuit (Runge et al., 1997)
MONTE CARLO ANALYSIS

Monte Carlo analysis was used to determine the parameter stability of these various models. The Monte Carlo boot strap technique used in this paper involves changing each experimental point by a randomly created, normally distributed deviation. The magnitude of the deviation is a function of the standard deviation of the experimental data point and is calculated using an algorithm outlined in Press (1992). Using this newly created data set, the model parameters can be re-calculated using the non-linear solving routines. By continually repeating this process, the distribution of the parameters is calculated. This distribution and its summary statistics, mean and standard deviation, are an approximation of the mean and standard deviation of the flotation parameter derived from the original experimental data (Press, 1992; Kocis, 1995). This process is represented diagrammatically in Figure 4.

The standard deviation of each parameter is a measure of how well defined the parameter is, based on the error in the experimental data set from which it is derived. To estimate the standard deviation of the parameters of the Red Dog floatability component models, a Visual Basic program embedded in Microsoft Excel was written to calculate new experimental data sets, to model fit these new data sets and to record the parameter sets derived from these new data sets. The program repeated this process 1000 times for each modelling exercise.

Figure 5 shows the frequency of the fast flotation rate constant of the two component model derived during the Monte Carlo analysis plotted as a function of the number of batch tests used in the fitting process. The results indicate that, as the number of tests used to derive the floatability components is increased, the confidence interval for each parameter decreases in size. The fast flotation rate constant is only defined sharply when at least four laboratory flotation tests are used during parameter derivation. The most interesting result of this analysis is that the parameter derived using only one set of data has a very large spread of results. This is the traditional method of calculating the component flotation parameters. This analysis shows that the parameters derived from this approach are not well defined.
As expected, parameter stability deteriorates as the number of components used in the analysis increases. In Figure 6, the frequency of derivation of one of the flotation rate constants derived from a single batch test is plotted as a function of the number of components used in the analysis (This plot has been truncated at a flotation rate of 1.0 to enable direct comparison between all graphs). It demonstrates how the stability of the parameter becomes quite poor (i.e. large spread in results) when multiple components are used during fitting.

This decrease in parameter stability when multiple components are used in the analysis even occurs when six batch laboratory tests are used in the fitting process (Figure 7). However, the spread in the results at higher parameterisation is significantly decreased.

**CONCLUSIONS**

One laboratory flotation test contains insufficient data to derive meaningful values of the parameters used in multiple floatability component modelling.

Inherent floatability is a property related to the stream in a flotation circuit that is independent of the operating conditions in the flotation units in the process. A previous study, using the Red Dog data set presented in this paper, showed that this property does not change throughout the flotation circuit and is conserved around nodes in a flotation process. Using these principles, floatability component modelling can utilise flotation data collected from laboratory tests of the different streams in a flotation process (Harris, 1997).

When additional data sets are used in parameter estimation and the problem is constrained so that the flotation rate in all tests is constant and the mass of floatability components are conserved around units, the stability of the parameters derived is substantially increased and these parameters can be potentially used for meaningful simulation.

**FUTURE WORK**

The actual ore floatability of a stream is highly complex due to the diverse nature of its particles. Can a simplified two or three discrete floatability component model successfully represent this more complex distribution and predict the performance of a circuit subject to a change?
The predictive ability of the discrete floatability component approach has also been studied by the authors and will be the topic of a future paper.

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