

## VISCOSITY AT HIGH SHEAR RATE MODEL IN MINERAL SUSPENSIONS

M. Oswaldo Bustamante R.<sup>1</sup> & Arturo Barrientos R.<sup>2</sup>

<sup>1</sup>Universidad Nacional de Colombia - Sede Medellín – Facultad de Minas - CIMEX – A.A. 1027, Medellín – Colombia - e-mail: [mobustam@perseus.unalmed.edu.co](mailto:mobustam@perseus.unalmed.edu.co)

<sup>2</sup>Universidad de Concepción - Depto. Metalurgia – Concepción – Chile, Casilla 53C Concepción – Chile e-mail: [abarrient@udec.cl](mailto:abarrient@udec.cl)

### ABSTRACT

The viscosity at high shear rate of suspensions composed of a Newtonian liquid and solids with size distribution were modeled from experimental data which were obtained from cup-and-bob viscometer in laminar flow region. The solids were coal, sulfur and quartz. The viscosity model is a function of the Rosin-Rammler characteristic size and distribution moduli of the particle size distribution and of the solid volumetric concentration.

The new viscosity model is a asymptotic solution of the constitutive equation for stress tensor that were developed by Bustamante and Barrientos (2000).

### INTRODUCTION

The modelling of the viscosity at high shear rate of the suspensions composed of a Newtonian liquid and particles with size distribution is some of the more important aspects in the understanding of the mechanical energy dissipation in rheology of the slurries.

Currently there exist many models of the suspension viscosity, and these models are give in terms of the volumetric fractions and one characteristic size of the solids in the suspensions.

To determine the characteristic size is not a problem when the solid material in the suspension is a monosize, whereas several characteristic sizes can be defined when the solid material is made of different size particles.

Tangsathikulchai & Austin (1990) defined as characteristic size the Rosin-Rammler size parameter, while Barrientos et. al (1994) used as characteristic size the average size in the size distribution.

On the other hand, only a characteristic size is not sufficient to get a good understanding about the role of the sizes distribution in rheology suspension.

The particles packing and clustering effects in mechanical dissipation energy, are difficult to understand if not have a parameter in the models that provide information about of the how is the size distribution in the suspension.

Bustamante (2000) developed a new model of the stress tensor for mineral suspensions and one asymptotic solution out of this model is a new viscosity model, when the stress tensor is applied to high shear rate.

This paper presents the new model of the suspension viscosity and this model takes into account clustering (Quemada, 1985) and self-diffusion effects (Leighton and Acrivos, 1987, Morris and Brady, 1996) in the rheologic behavior of the mineral suspensions.

### THEORY

The viscosity prediction in suspensions with solids that have a size distribution, is not easy. Some works show the size effects over the viscosity of the slurries, Dabak and Yucel (1987), Farris (1968), Hopkins and Woodcock (1990), Kim and Luckham (1993), Tangsathikulchai and Austin (1990), Alejo (1992), Barrientos et. al. (1994), Bustamante (2000) developed a constitutive equation of the stress tensor in mineral suspension:

$$\tau = \tau_f \left[ \exp \left[ A \cdot \exp \left\{ \frac{\phi^{s(m)}}{1 - B \phi^{s(m)}} \right\} \right] \right] \quad (1)$$

with:

$$A = \alpha_0 + \alpha_1 \ln Re + \alpha_2 \ln Pe + \alpha_3 \left( \ln \frac{De}{Fr} \right)^3 \quad (2)$$

$$B = \tilde{B}(m)$$

where  $Re$ ,  $Pe$ ,  $Fr$  and  $De$  are Reynolds, Peclet Froude and Deborah numbers respectively.

Each one of the numbers, is defined as:

$$Re = \frac{\rho_f K_{RR}^2 \dot{\gamma}}{\mu_f}, \quad Fr = \frac{\dot{\gamma}^2 K_{RR}}{g} \quad (3)$$

$$Pe = 3\pi\mu_f \frac{\dot{\gamma}}{KT}, \quad De = \frac{\mu_f}{\rho_s K_{RR}^2 \dot{\gamma}}$$

where  $\phi$  is the solid volumetric fraction,  $\rho_f$  and  $\rho_s$  are the fluid and solid density, respectively,  $\mu_f$  is the fluid viscosity,  $K_{RR}$  and  $m$  are defined in the equation (4), and  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_3$  are parameters that depend on the mineral and  $\dot{\gamma}$  is the shear rate.

The equation (1) is the model of the shear stress for a suspension composed of a Newtonian liquid and minerals with a size distribution, characterized by the empirical Rosin-Rammler equation:

$$F(x) = 1 - \exp \left[ - \left( \frac{x}{K_{RR}} \right)^m \right] \quad (4)$$

where  $F(x)$  is the cumulative fraction less than size  $x$ ,  $K_{RR}$  is the fineness modulus and  $m$  is the distribution modulus.

Working out the equation (1) for  $\dot{\gamma} \rightarrow \infty$  and dividing by shear rate, it is possible to obtain a viscosity expression for high shear rate, that is, the Newtonian viscosity at high shear rate.

The new model is:

$$\ln \left[ \frac{\mu_{sp}}{\mu_f} \right] = P(K_{RR}) \exp \left[ \frac{\phi^{g(m)}}{1 - B\phi^{g(m)}} \right] \quad (5)$$

$P(K_{RR})$  is a function of the Rosin-Rammler size parameter, while  $g(m)$  was deduced by Bustamante (2000):

$$g(m) = 1 - \frac{2}{3m} \quad (6)$$

with  $m$  as was defined in the equation (4).

Bustamante and Barrientos (2000) developed a viscosity model under a termomechanical approximation with an equal structure of the model presented in the equation (5).

## EXPERIMENTAL

The rheological properties of the suspensions were determined in the laminar region using Haake Rotoviscosimeter RV20. All experiments were carried out at a constant temperature by using a heating-refrigerating bath and circulator system to maintain the temperature at  $11 \pm 0.5$  °C.

The minerals used were crystalline quartz, coal-coke and sulfur.

### Granulometrical

The table I shows the size distribution parameters for each one of the minerals used.

Table I. Rosin-Rammler parameters of the mineral size distribution.

MINERAL	$K_{RR}$	$m$
	Microns	
Sulfur	23.07	1.20
	33.37	1.20
	102.00	3.20
Crystalline quartz	4.64	1.105
	27.90	1.106
	368.60	1.107
Coke-coal	158.70	0.89

The liquids used to prepare the slurries were mixtures of aliphatic naphthas and sodium polytungstate to match the density solids. The liquid is Newtonian over most of shear rate range. All tests were run in laminar flow region (Tangsathitkulchai and Austin, 1990).

This liquids used in making suspensions also were apolares and with equal density of the minerals, therefore, electroviscous and gravitational effects. were avoided in the suspensions viscosity determination.

## RESULTS

The figures 1, 2 and 3 show the experimental data and simulated values of the reduced viscosity for sulfur and crystalline quartz and coal respectively.

In figure 1, the change of the parameter  $m$  in the distribution is very large and, however, the  $B$  parameter is constant, that is, The  $B$  parameter no depend on the mineral distribution, this depend of the mineral only.

In figure 2, the change of the  $K_{RR}$  parameter and the effect on the viscosity are shown. This parameter has effects over  $P(K_{RR})$  only, whilst it has no effect over  $B$  parameter.

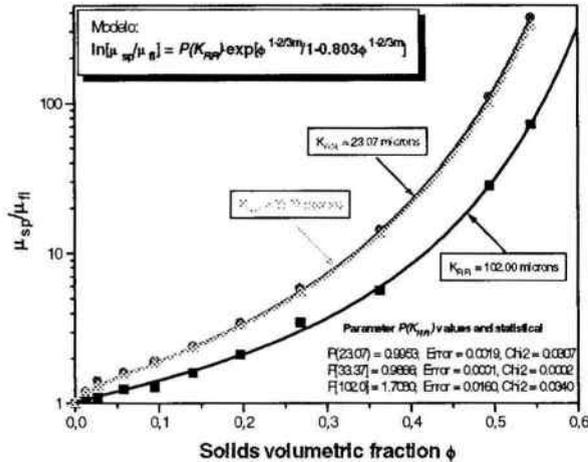


Figure 1. Experimental and simulated data for sulfur suspensions of the reduced viscosity.

Figure 3, shows the model scope for predicting viscosity when the mineral density suffers a drastic change and when the mineral shape is very irregular.

For all test, the error for the  $B$  parameter is less than 0.001, and  $Chi^2$  is near to zero.

Further, the parameter  $B$  in the model is near to one.

DISCUSSION

Viscosity models that take into account the particle size distribution are not many. Tangsathitkulchai and Austin (1990), Alejo (1992), Barrientos et. al (1994) and Bustamante and Barrientos (2000) developed models considering size distribution of the solids in the slurry.

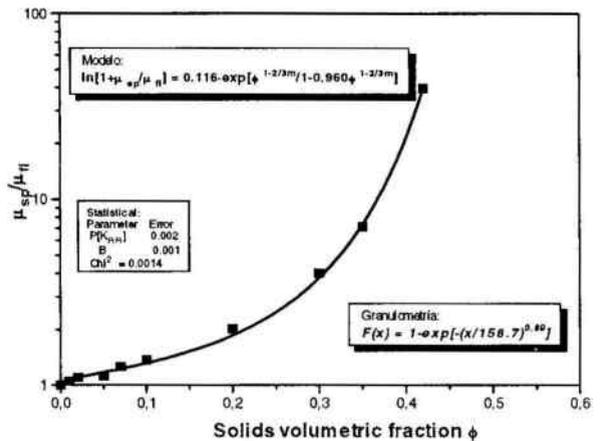


Figure 3. Reduced viscosity of coal, experimental data and simulated curve.

The Rosin-Rammler equation has two parameters,  $K_{RR}$  for the characteristic size and  $m$  to characterize the distribution.

The packing in the suspensions are more related to  $m$  parameter than to  $K_{RR}$  parameter. However, Tangsathitkulchai and Austin (1990) found that  $m$  parameter is not as important as  $K_{RR}$  parameter, when they worked on natural distribution from ball mills.

In this sense, Bustamante (2000) found that  $m$  parameter is important in the control of the particle packing conditions, but  $K_{RR}$  controls the cluster size. Therefore, both parameters are important in rheology suspensions as determined by Alejo (1992), Barrientos et. al (1994), Hopkins and Woodcock (1990) and Bustamante and Barrientos (2000).

However, Tangsathitkulchai and Austin's data reproduce a minimum viscosity, similar to Bustamante's data (Bustamante, 2000) where the suspensions are formed by solid particles with unimodal size distribution, and where the  $m$  parameter has a little effect over the viscosity (see figure 4). Under other conditions, with two or more modes in the size distribution the viscosity increase and the role of the  $m$  parameter is more important.

The model proposed in this paper takes into account the size effect over the viscosity and the form

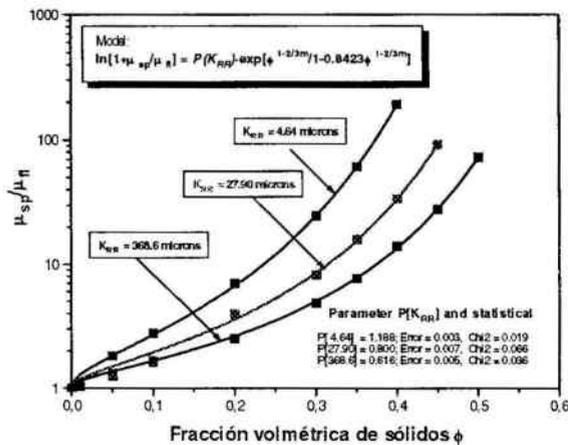


Figure 2. Reduced viscosity for crystalline quartz changing distribution parameter  $m$ .

of size distribution through  $m$  parameter. This model apart sets size effect and distribution effect.

On the other hand, this model is valid for suspensions with monosize particles. In this case,  $m \rightarrow \infty$  and the function  $g(m) \rightarrow 1$ , and therefore, the polydispersity effect is minimal and the viscosity is increased, as Quemada (1985) presented before.

Further it is possible to obtain this model by means of a termomechanical approach as Bustamante and Barrientos (2000) obtained it.

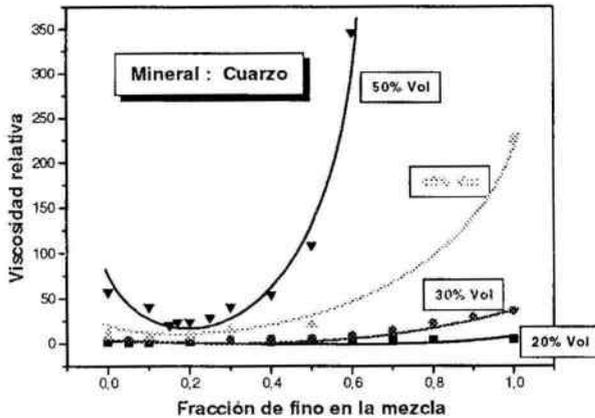


Figure 4. Effects of addition of fines over viscosity changing from unimodal (minimum) a multimodal size distribution.

## CONCLUSIONS

This paper presents a new model of the slurry reduced apparent viscosity.

In this model it is possible to set size effect and distribution effect over rheologic behavior of the suspensions.

The  $B$  parameter in this model (see equation 5) depend neither from the characteristic size  $K_{RR}$  nor from the distribution parameter  $m$ . It depends only on the mineral or possibly on the particle shape.

## ACKNOWLEDGEMENTS

This work was partly funded by Universidad de Concepción, The Oficina de Cooperación Iberoamericana (Becas Mutis) and partly by the Universidad Nacional de Colombia –Sede Medellín.

## REFERENCES

- Alejo, B. Size distribution and concentration effects over the viscosity of the suspensions. Magister thesis, U. of Concepción (in Spanish), 1992.
- Barrientos, A., Concha, F. And León, J. L., A Mathematical model of solid-liquid suspensions, IV Meeting Southern Hemisphere on Mineral Technology, Concepción – Chile, 189-202 (1994).
- Bustamante, O. and Barrientos A., A suspension Model under Termomechanic Approach, Encuentro argentino de tratamiento de minerales (In Spanish), (200).
- Dabak, T and Yucel O, Modeling of concentration and particle size effects on the rheology of highly concentrated suspensions. Power Technology, 52:193-206 (1987).
- Farris, R. J. Prediction of the viscosity of multimodal suspensions from unimodal viscosity data, Chem. Eng. Sci., 23: 895-899 (1968).
- Hopkins, A. and Woodcock, L., Effects of polydispersity on osmotic properties of colloidal suspension, J. Chem. Soc. Faraday Trans., 86(20):3419-3428 (1990).
- Kim T. and Luckham P. F., Some rheological properties of bimodal size particles, Power Technology, 77:31-37 (1993).
- Leighton, D. and Acrivos, A., Shear induced migration on concentrated suspensions, J. Fluid Mech., 415-439 (1987).
- Morris, J. and Brady, J. F. Self-diffusion in sheared suspensions, J. Fluid Mech., 312:223-252 (1996).
- Quemada, D., Phenomenological rheology of concentrated dispersions, I. Clustering effects and structure-dependent packing fraction, J. Theoretical and Applied Mechanics, 267-288 (1985).
- Storms, R., F., Ramarao, B., Weiland, R. H., Low shear rate viscosity of bimodally dispersed suspensions, Power Technology, 63:247-259 (1990).
- Tangsathitkulchai, C and Austin, L. Rheology of concentrates slurries of particles natural size distribution produced by grinding, Power Technology, 56, 293-299 (1990).