APPLICATION OF THE UNIFIED MODEL OF SOLID-LIQUID SEPARATION OF FLOCULATED SUSPENSIONS TO EXPERIMENTAL RESULTS

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ABSTRACT

We apply the unified model of solid-liquid separation of flocculated suspensions, presented in a companion contribution, to three published experimental studies of batch sedimentation, centrifugation, and pressure filtration with sedimentation, respectively. The necessary constitutive equations are adopted from the findings of the respective authors of these works. The numerical simulations are compared with the experimental results, and illustrate important features of the unified model.

INTRODUCTION

In this paper we apply the unified mathematical theory of solid-liquid separation outlined by Bürger et al. (2001) to selected experiments from the literature. To this end, we review three theoretical and experimental studies of sedimentation, centrifugation and pressure filtration. After a brief account of the theory employed in each case to interpret the measurements, we apply our phenomenological model to the available data. The two necessary constitutive functions, the Kynch batch flux density function \( f = f(\phi) \) and the effective solid stress function \( \sigma_s = \sigma_s(\phi) \), are determined from the published concentration, permeability and compressibility data. The mathematical model is then solved numerically using these functions. The resulting simulations of solid-liquid separation behaviour are compared to the respective authors' experimental findings and interpretations.

Comparisons of the phenomenological model with experimental results have already been performed for batch settling experiments (Bürger et al. 1999, 2000b; Garrido et al. 2000). The novelty of this contribution consists in the recalculation of new experiments.

CASE 1: SIMULATION OF A BATCH SETTLING EXPERIMENT

Consider first an experiment of batch sedimentation of flocculated suspension. For that application, the unified mathematical model of solid-liquid separation reduces to the field equation (7) together with the initial and boundary conditions (11) and (12) of Bürger et al. (2001).

The recalculated sedimentation experiment is that of the settling of an attapulgite suspension as reported by Tiller and Khatib (1984). This experiment was also considered by Diplas and Papanicolaou (1997) as a test case for their numerical model. The physical constants of the suspension were the solid and fluid mass densities \( \rho_s = 2300 \, \text{kg/m}^3 \) and \( \rho_f = 1000 \, \text{kg/m}^3 \), the initial concentration \( \phi_0 = 0.03 \) and the height of the suspension \( L = 0.4 \, \text{m} \).

We made first numerical experiments with the following effective stress function, which were obtained from Eqns. (1f) and (1h) of Diplas and Papanicolaou (1997) by inserting constants given in their Table 1:

\[
\phi_s(\phi) = 0.065 + 1.1 \times 10^{-6} \phi,
\]

\[
\sigma_s(\phi, t) = \begin{cases} 
0 & \text{for } \phi \leq \phi_s(t) \\
153.14 \left[ \left( \frac{\phi}{\phi_s(t)} \right)^{2.77} - 1 \right] & \text{for } \phi > \phi_s(t).
\end{cases}
\]

With this function, we obtained very small concentration changes within the sediment bed. In particular, the final concentrations obtained at the bottom of the vessel were much lower than the values, roughly in the range from 0.12 to 0.14, obtained in the simulations by Diplas and Papanicolaou (their Figure 6). To change this, note first that by Eq. (12) of Bürger et al. (2001), the boundary condition at \( z = 0 \) for batch settling in a column can be rewritten as...
\[ \frac{\partial \phi}{\partial z} = -\delta_{x} g \phi / \sigma_{c} (\phi) \quad \text{at} \quad z = 0, \quad t > 0, \]  
\hspace*{2cm} (3)
i.e. the concentration gradient depends essentially on the derivative of the effective solids stress function. Therefore, to produce steeper concentration profiles at \( z = 0 \), we replaced the derivative of Eq. (2) by

\[ \sigma_{c} (\phi, t) = \begin{cases} 0 & \text{for} \quad \phi < \phi_{c} (t), \\ 153.14 \frac{\partial}{\partial \phi} \left( \left( \frac{\phi}{\phi_{c} (t)} \right)^{2} - 1 \right) \text{Pa} & \text{for} \quad \phi > \phi_{c} (t), \end{cases} \]  
\hspace*{3cm} (4)
where \( \phi_{c} (t) \) is defined in Eq. (1) and Figure 1 shows the function

\[ n (\phi) = \begin{cases} 2.77 & \text{for} \quad \phi_{c} (t) \leq \phi \leq 0.09, \\ 0.75 + 202 (0.1 - \phi) & \text{for} \quad 0.09 \leq \phi \leq 0.01, \\ 0.75 & \text{for} \quad \phi > \phi_{c} (t). \end{cases} \]  
\hspace*{3cm} (5)

Figure 1: Effective solid stress function \( \sigma_{c} = \sigma_{c} (\phi, t) \) for the simulation of the settling of an attapulgite suspension (Tiller and Khatib 1984, Diplas and Papanicolaou 1997).

Using the well-established formula (Bürger et al. 2000b) for the conversion of permeability values into values of the flux density function,

\[ f (\phi) = -K (\phi) \delta_{x} g \phi^{2} / \mu_{f}, \]  
\hspace*{3cm} (8)
where \( \mu_{f} \) is the dynamic viscosity of the fluid, and assuming \( \phi_{c} = 0.065 \) and \( \mu_{f} = 0.001 \) Pa s, we obtain the expression

\[ f (\phi) = -1.166 \times 10^{-11} \phi^{-1.4883} \text{ m/s}, \]  
\hspace*{3cm} (9)
which was evaluated to generate the data points for \( f (\phi) \) for \( \phi \) larger than 0.17. The resulting flux density function \( f (\phi) \) having three inflection points is not unreasonable, since similar flux density functions were already considered by Scott (1968) and more recently by Font et al. (1998) (cf. their Figure 2).
Figure 3: Numerical simulation of batch sedimentation of an attapulgite suspension: settling plot with isoconcentration lines (top) and concentration profiles (bottom). The symbols (o and O) correspond to interfaces measured by Tiller and Khatib (1984).

Figure 3 shows the result of our numerical simulation with these parameters, performed with the upwind difference method described in detail by Bürger and Karlsen (2001) and a spatial discretization of $\Delta z = L/300$. Concerning the settling and similar filtration plots shown in this contributions it should be mentioned that over an interval of concentration values will always appear as a cumulation of the numerical isolines of the concentrations contained in that interval.

**CASE 2: SIMULATION OF A BATCH CENTRIFUGATION EXPERIMENT**

We next solve numerically one-dimensional solid-liquid separation model for batch centrifugation of a flocculated suspension in a rotating tube, as given by Eqns. (9) with $\gamma = 0$ and the initial and boundary conditions, Eqns. (14) and (15), of our companion paper (Bürger et al. 2001). To this end, we consider the study by Eckert et al. (1996), who present centrifugal experiments conducted to quantify the sedimentation and consolidation behaviour of fine tails generated by the extraction of bitumen from certain tar sands. In that paper, the applied angular velocity $\omega$ is increased successively to increase compression. These authors explicitly state the following constitutive functions for their material:

$$f(\phi) = -7.04 \times 10^{-3} \phi (1-\phi)^{4.03} \text{ m/s}, \quad (10)$$

$$\sigma_c(\phi) = \left\{ \begin{array}{ll} 0 & \text{for } \phi \leq \phi_c, \\ 3.056 \times 10^6 \phi^6 & \text{for } \phi > \phi_c. \end{array} \right. \quad (11)$$

A critical concentration is not proposed, but the published experimental information, in particular the observed behaviour of the suspension/sediment interface, suggest that $\phi_c$ should be close to the initial concentration $\phi_i = 0.14$. We therefore chose $\phi_c = 0.14$. Unfortunately, neither the outer radius of the rotating sedimentation tube $R$ (only the height of the suspension 0.047m), nor the solid-liquid density difference $\Delta \rho$ were published.

In the context of a different simulation of a settling experiment with the same material, Eckert et al. (1996) mention that “the final predicted concentration of solids at the bottom of the sediment was 25% by volume or 58% by weight”, from which, by assuming $\rho_f = 1000 \text{ kg/m}^3$, one can conclude that $\rho_s = 4143 \text{ kg/m}^3$. We believe that this value is too high and doubt that the indicated 25% concentration value is correct: Figure 16 of Eckert et al. (1996), to which the cited comment refers, clearly shows how a suspension of initial concentration $\phi_i = 0.07$ forms a sediment layer whose thickness is a bit less than a quarter of the initial height. Therefore the average solids concentration of the sediment should equal roughly 0.28. Since the concentration at the bottom of the sediment is higher than at the sediment/suspension interface, the bottom concentration at the bottom should be at least 28%.

In the simulation we used $\rho_s = 2600 \text{ kg/m}^3$ and found best agreement for an outer radius $R = 0.1m$, which is reasonable for a laboratory centrifuge. The applied centrifugal force $R\omega^2$ varied between 8.3 and 305 g, where $g$ denotes the acceleration of gravity (here set to 9.81 m/s$^2$). Figure 4 shows the result of our numerical simulation, which was obtained by an application of the appropriate modification of the upwind finite difference method, see Bürger and Karlsen (2000b) and Bürger and Concha (2001) for details. Note that although the initial concentration coincides with the critical, a stress-free suspension zone is forming, as can
be seen from the formation of vertical iso-concentration
lines for $\phi = 0.135, 0.13$ and $0.125$.

![Diagram](image)

Figure 4: Numerical simulation of batch centrifugation of a suspension: settling plot (top) and concentration profiles (bottom). The open circles (o) refer to the supernate/suspension interface measured by Eckert et al. (1996).

**CASE 3: SIMULATION OF AN EXPERIMENT OF PRESSURE FILTRATION WITH SIMULTANEOUS SEDIMENTATION**

We finally apply the unified model of solid-liquid separation to experimental data for pressure filtration. In that case, the relevant equations of Bürger et al. (2001) are the field equation (7), the kinematic boundary conditions (16) and (17) and the dynamic boundary condition

$$\sigma(t) = \sigma_s(\phi(0,t)) - g[m + \rho_f(h(t) - h_0)] - \mu_f R_m h(t), \quad (12)$$

We recall that $\sigma(t)$ is the applied pressure, $m$ the initial suspension mass, $h(t)$ the piston height, $h_0$ the initial piston height, $\mu_f$ the dynamic viscosity of the fluid, and $R_m$ the resistance of the filter medium.

Figure 5: Kynch batch flux density function $f(\phi)$ determined from experimental information by Font and Hernández (2000).

Here we consider a recent study by Font and Hernández (1999), who performed experiments with a calcium carbonate suspension having the parameters $\rho_s = 2648 \text{ kg/m}^3$, $\mu_f = 1000 \text{ kg/m}^3$, and $\phi_0 = 0.1115$.

For $0.16 < \phi \leq 0.22$, we adopted the respective expression $f(\phi) = f_3(\phi)$ and $f(\phi) = f_3(\phi)$ stated explicitly Font and Hernández (2000), with the modification that the coefficient $-3.76 \times 10^4$ in their equation (13) was replaced by $-3.83 \times 10^4$ to make the flux density function continuous. The segment $f(\phi) = f_3(\phi)$, valid for $0.22 < \phi \leq \phi_{\text{max}} = 0.6$, was designed in such a way that it connects smoothly with $f_3(\phi)$, and that the resulting permeability value at $\phi = \phi_{\text{max}}$ is consistent with that obtained from the average specific cake resistance formula, Eq. (20) in Font and Hernández (2000).

We directly adopted the effective solid stress formula given in that paper,

$$\sigma_s(\phi) = \begin{cases} 
0 & \text{for } \phi < \phi_c = 0.25, \\
1.279 \times 10^9 \phi^{2.176} \text{Pa} & \text{for } \phi > \phi_c 
\end{cases} \quad (13)$$

where the chosen value of $\phi_c$ is in the range 0.215 to 0.27 considered appropriate in that paper.

The values of the applied pressure used by Font and Hernández (2000), defined as the pressure differences between the top of the suspension and the bottom of the filter medium, varied between 17 and 97 mmHg, i.e. between 2266.1 and 12930.1 Pa. The initial height was $h_0 = 0.063$ m. We simulated the experiment with $\sigma(t) = 57$ mmHg = 7598.1 Pa, for which detailed plots of observed interfaces were provided.
Figure 6: Pressure filtration with simultaneous sedimentation: filtration plot (top) and concentration profiles (bottom). The symbols (\(A, \phi\) and 0) represent interfaces measured by Font and Hernández (2000).

For first test calculations we used the parameters \(\mu_1 = 10^2\) Pa s and \(R_w = 3.87 \times 10^{10}\) m\(^{-1}\). It turned out, however, that the use of these values produced instabilities of the numerical method. These became apparent as a consequence of the direct iteration adopted from Bürger et al. (2000a), which has proved to work well for large applied pressures, in the right-hand part of the equation

\[
\sigma_{\phi}(\phi(0,t)) = \sigma(t) + g[m + \rho_s(h(t) - h_0)] + \mu_1 R_w h(t),
\]

which became negative such that it could not be inverted to yield the value \(\phi(0,t)\). Here, we circumvented this problem by using the published pressure drops across various height differences of the system. In particular, the pressure difference across the filter medium could be calculated from Font and Hernández' Table 3. For the run considered here, this difference, denoted here by \(\sigma_{\phi}\), is 5 mmHg or 666.5 Pa. We therefore neglected the effect of filter medium resistance, i.e. utilized the dynamic boundary condition

\[
\sigma_{\phi}(\phi(0,t)) = \sigma(t) - \sigma_{\phi} + g[m + \rho_s(h(t) - h_0)].
\]

Figure 6 shows our numerical simulation of the experiment, obtained with the numerical method outlined in Bürger et al. (2000a) (which is yet another version of the upwind finite difference scheme) with a discretization \(\Delta z = h(t)/200\). Note that the movement of the piston makes it necessary to adapt the mesh in each time step. The concentration profile diagram also displays a series of fat black dots, which denote the concentration just below the piston height. The series of dots illustrates that very quickly a clear liquid zone forms between the bulk suspension and the piston.

**DISCUSSION**

The test cases presented here illustrate several features of the unified mathematical model of thickening, filtration and centrifugation of flocculated suspensions.

All three examples illustrate the ability of the model to correctly predict the piston height (in the case of pressure filtration) and the suspension/supernate and filter cake/suspension interfaces, without bearing the necessity to explicitly track and switch between different equations across them. This contrasts in particular with the significantly more complicated treatments of Diplas and Papanicolaou (1997); the authors of that paper essentially use the same model as that outlined in this paper, but use separate solution procedures for the hyperbolic and the parabolic compression part, which requires a manual graphical procedure and tracking of the type-change interface, i.e. the sediment level.

Moreover, in the first and the third case of this paper, we observe that fans of characteristics, i.e. the sequences of iso-concentration lines between 0.031 and 0.035 in Figure 3 and from \(\phi = 0.115\) to \(\phi = 0.16\) in Figure 6, separate the bulk suspension from the sediment or filter cake. These characteristics appear to be emerging tangentially from the rising sediment or filter cake surface, corresponding to the iso-concentration line \(\phi = \phi_c\). It should be emphasized that these tangential characteristics have been determined numerically, and that their presence is not part of the a priori assumptions of our mathematical model. This contrasts with the explicit assumption of the existence of tangential characteristics going back to Fitch (1982), which also enters into both Diplas and Papanicolaou's and Font and Hernández' construction procedures. Since the presence of tangential characteristics is the key ingredient to some interesting applications (Font et al. 1999, Font and Laveda 2000), it would be interesting to analyze analytically under which conditions they form as part of the entropy solution of the strongly degenerate parabolic-
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hyperbolic field equation, removing the necessity to postulate them a priori.

The mentioned difficulties with the appropriate coupling condition in the case of pressure filtration, which were circumvented here, alert to the fact that the model formulation for pressure filtration, involving a free boundary problem, gives in some circumstances rise to numerical problems that still have to be solved.

The results illustrate the predictive power of the unified mathematical theory. However, we did not make the effort to seek the utmost degree of agreement of the simulation with provided numerical or experimental information, since the model functions \( f(\phi) \) and \( \sigma_{\phi}(\phi) \), enter in a nonlinear and coupled way into the solid-liquid separation model, and their independent determination such that the error between simulation and experiment is minimized is still an open, and important problem.

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REFERENCES


